
Rs Aggarwal
Class 10
Trigonometric Ratio
Exercise 10 Question 1 to 31
One Shot

Que 1:- If $\sin\theta = \frac{\sqrt{3}}{2}$, find the value of all T- ratios of θ

$\sin\theta = \frac{\sqrt{3}}{2}$

As we know that $\sin\theta = \frac{P}{H}$

$\therefore \sin\theta = \frac{AB}{AC}$ — (1)

$\therefore \sin\theta = \frac{\sqrt{3}x}{2}$ — (2)

$\Rightarrow \frac{AB}{AC} = \frac{\sqrt{3}x}{2x}$

By Pythagoras theorem $P^2 + B^2 = H^2$

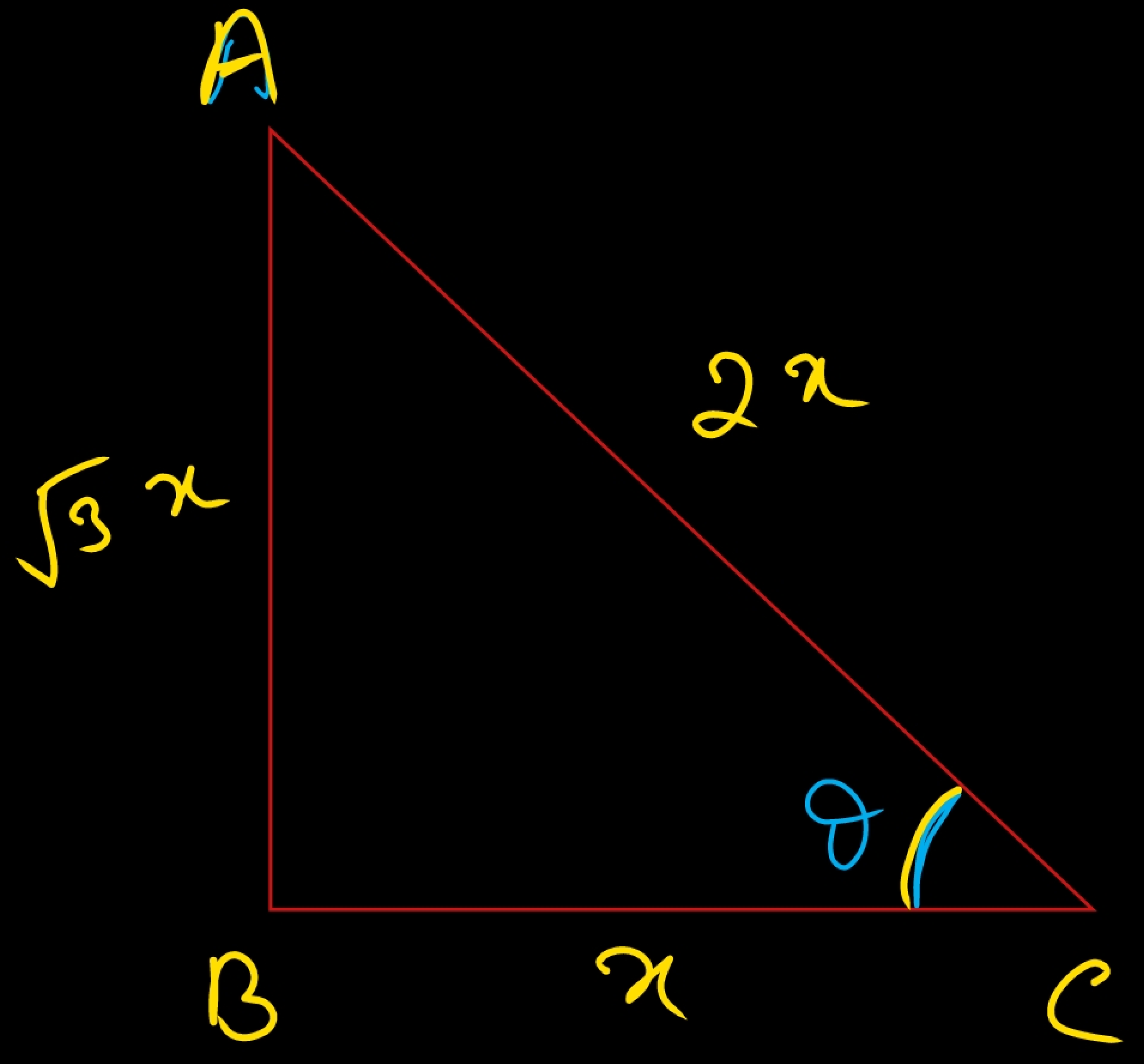
$\Rightarrow (\sqrt{3}x)^2 + BC^2 = (2x)^2$

$\Rightarrow 3x^2 + BC^2 = 4x^2$

$\Rightarrow BC^2 = 4x^2 - 3x^2$

$\Rightarrow BC = \sqrt{x^2}$

$\Rightarrow \boxed{BC = x}$



(1) $\sin\theta = \frac{\sqrt{3}\cancel{x}}{2\cancel{x}} = \frac{\sqrt{3}}{2}$

(2) $\cos\theta = \frac{1}{2}$

(3) $\tan\theta = \frac{\sqrt{3}\cancel{x}}{\cancel{x}} = \sqrt{3}$

$\cot\theta = \frac{1}{\sqrt{3}}$

$\sec\theta = 2$

$\operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$ Ans

Que 2:- If $\cos\theta = \frac{7}{25}$, find the value of all T-ratios of θ

$\cos\theta = \frac{7}{25}$ (given)

As we know that $\cos\theta = \frac{B}{H}$

$\therefore \cos\theta = \frac{7x}{25x}$ — (1)

$\therefore \cos\theta = \frac{BC}{AC}$ — (2)

from eqⁿ (1) & (2)

$\frac{BC}{AC} = \frac{7x}{25x}$

By Pythagoras theorem $P^2+B^2=H^2$

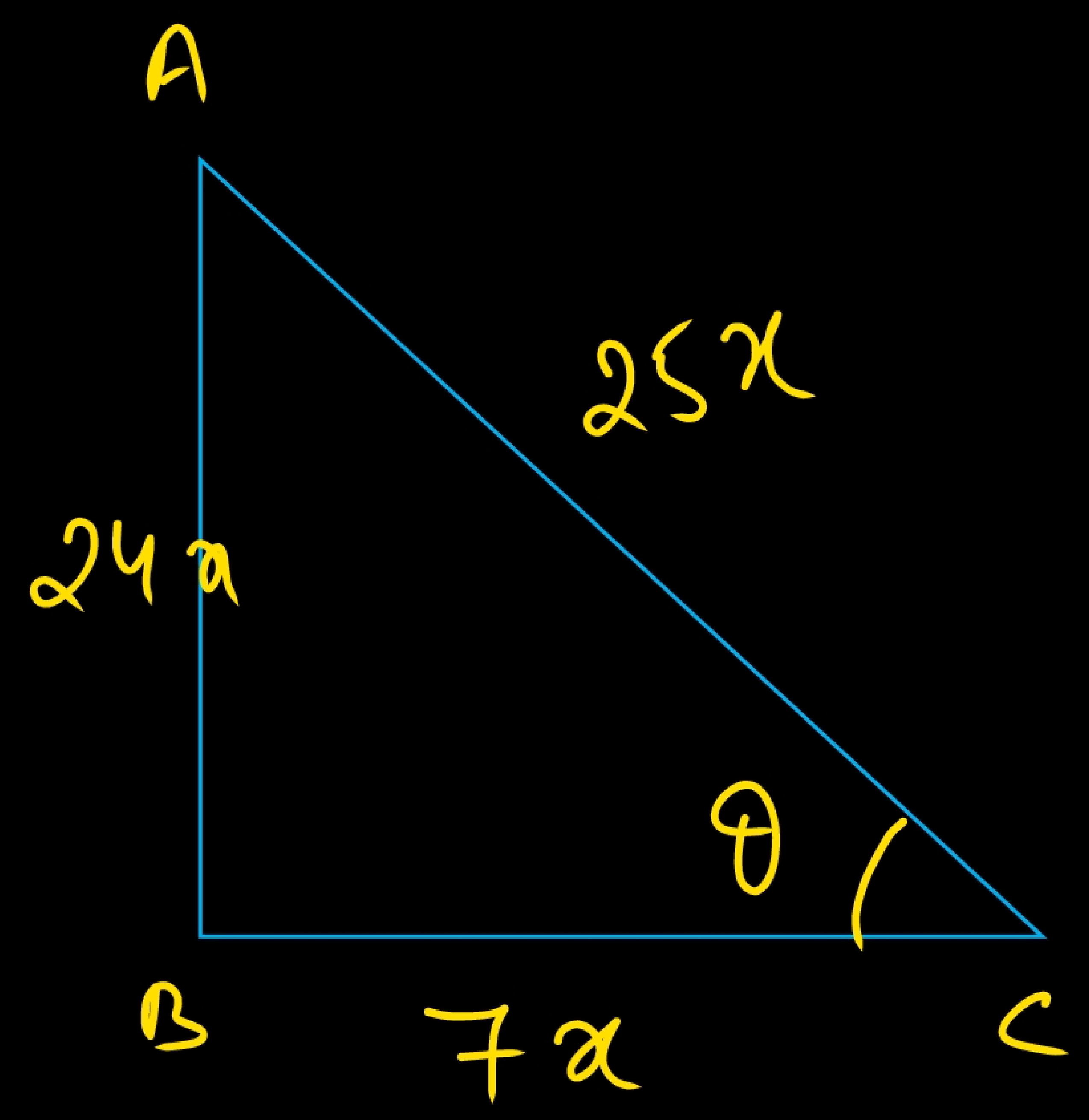
$\Rightarrow AB^2+(7x)^2=(25x)^2$

$\Rightarrow AB^2 + 49x^2 = 625x^2$

$\Rightarrow AB^2 = 625x^2 - 49x^2$

$\Rightarrow AB = \sqrt{576x^2}$

$\Rightarrow \boxed{AB = 24x}$



(1) $\sin\theta = \frac{24\cancel{x}}{25\cancel{x}} = \frac{24}{25}$

(2) $\cos\theta = \frac{7}{25}$

(3) $\tan\theta = \frac{24}{7}$

(4) $\cot\theta = 7/24$

(5) $\sec\theta = 25/7$

(6) $\operatorname{cosec}\theta = \frac{25}{24}$

Que 3:- If $\tan\theta = \dfrac{15}{8}$, find the values of all T-ratios of θ

As we know that $\tan\theta = \dfrac{P}{B}$

$\therefore \tan\theta = \dfrac{AB}{BC}$ — (1)

Now, $\tan\theta = \dfrac{15x}{8x}$ — (2)

from eqn (1) & (2)

$\therefore \dfrac{AB}{BC} = \dfrac{15x}{8x}$

By the Pythagorous theorem we know that

$P^2 + B^2 = H^2$

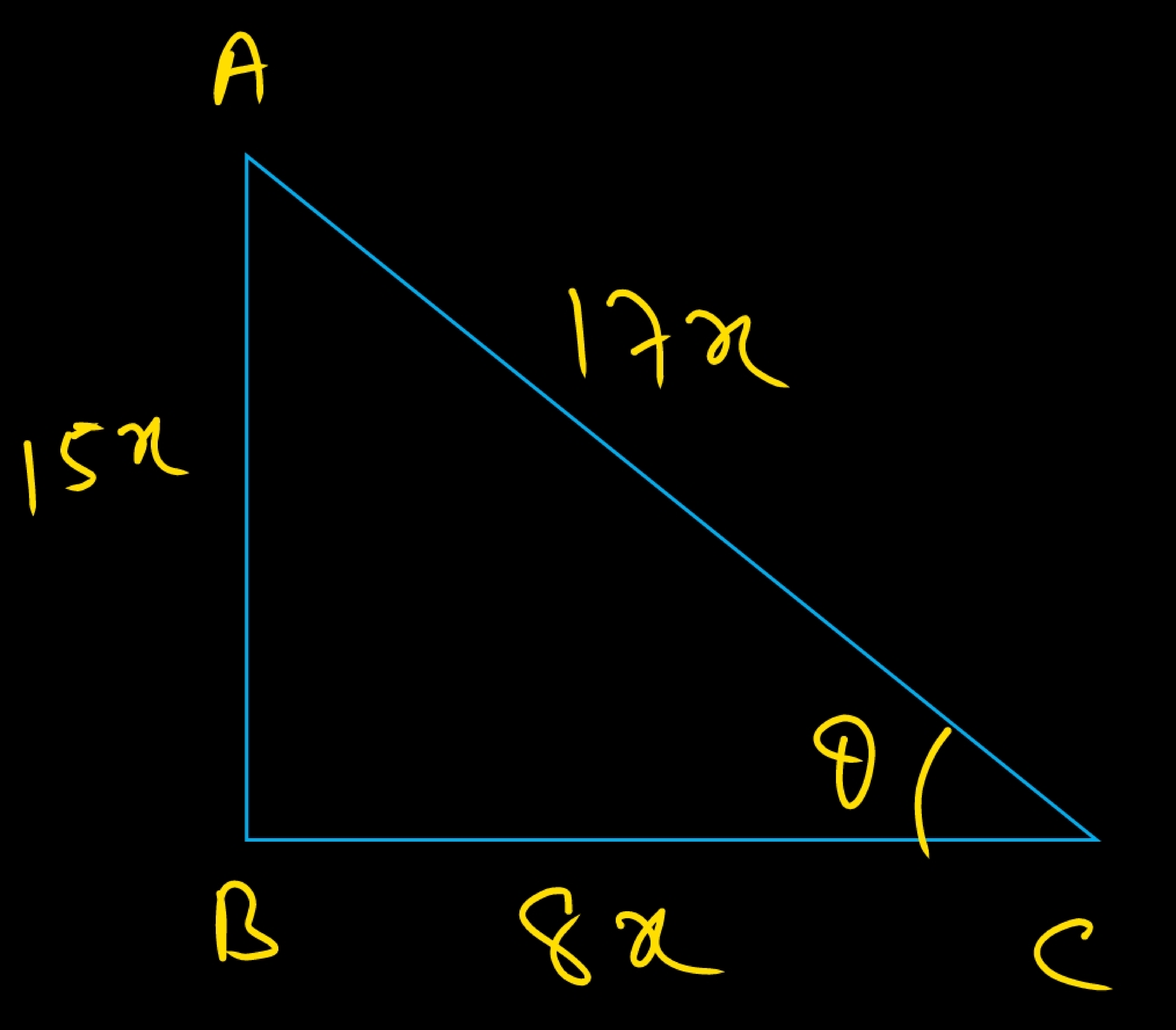
$\Rightarrow AB^2 + BC^2 = AC^2$

$\Rightarrow (15x)^2 + (8x)^2 = AC^2$

$\Rightarrow 225x^2 + 64x^2 = AC^2$

$\Rightarrow \sqrt{289x^2} = AC$

$\Rightarrow \boxed{17x = AC}$



(1) $\sin\theta = \dfrac{15\cancel{x}}{17\cancel{x}} = \dfrac{15}{17}$

(2) $\cos\theta = \dfrac{8}{17}$

(3) $\tan\theta = \dfrac{15}{8}$

(4) $\cot = \dfrac{8}{15}$

(5) $\sec\theta = 17/8$

$\csc\theta = \dfrac{17}{15}$ Ans

Que 4:- If $\cot\theta = 2$ find all the values of all T-ratios of θ

As we know that $\cot\theta = \frac{B}{P}$

$\therefore \cot\theta = \frac{BC}{AB}$ — (1)

$\cot\theta = \frac{2x}{1x}$ — (2)

$\Rightarrow \frac{BC}{AB} = \frac{2x}{1x}$

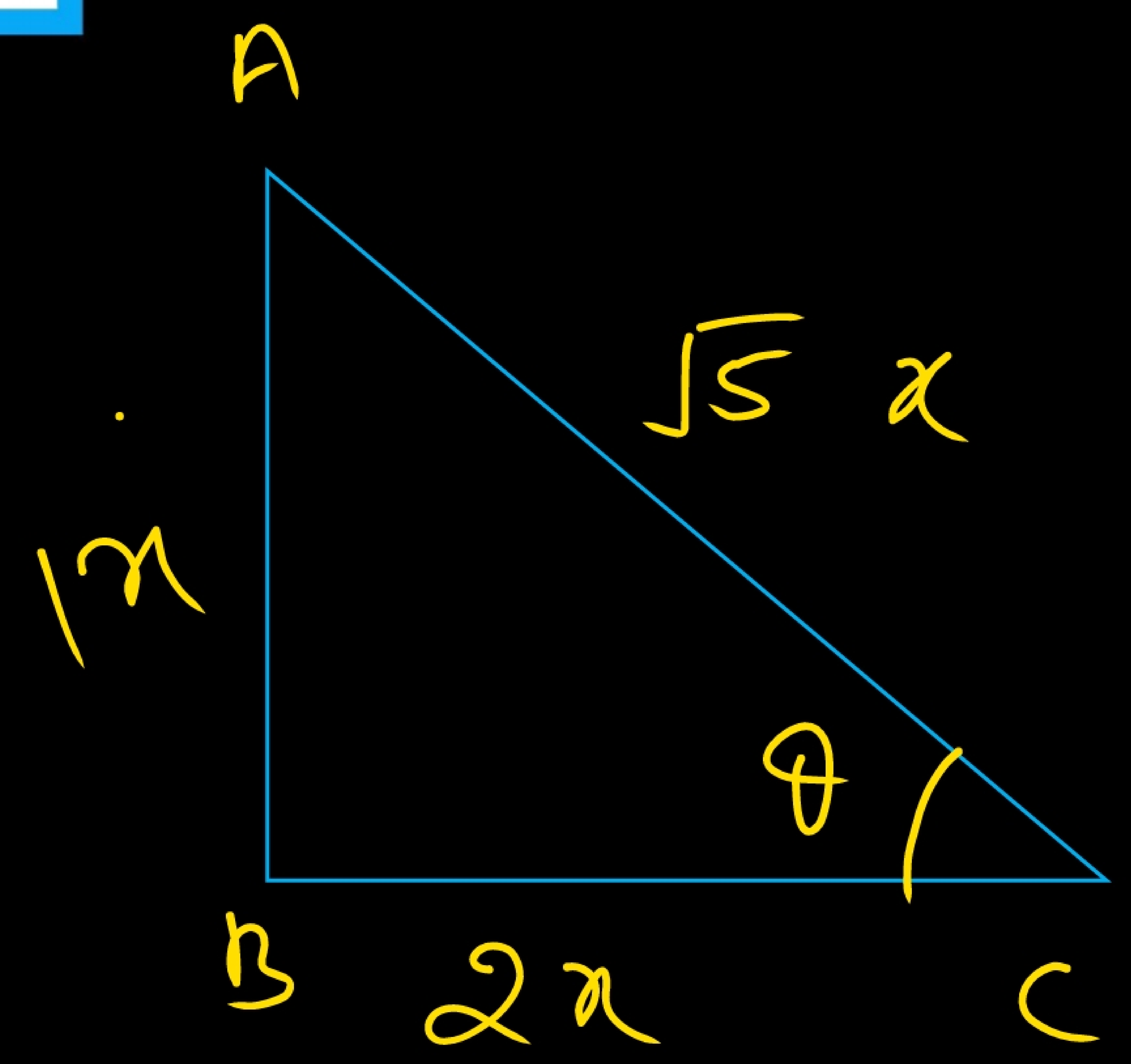
By pythagorous theorem $\boxed{P^2 + B^2 = H^2}$

$\Rightarrow (1x)^2 + (2x)^2 = AC^2$

$\Rightarrow x^2 + 4x^2 = AC^2$

$\Rightarrow AC = \sqrt{5x^2}$

$\Rightarrow AC = \sqrt{5}\,x$



(1) $\sin\theta = \frac{1x}{\sqrt{5}x} = \frac{1}{\sqrt{5}}$

(2) $\cos\theta = \frac{2x}{\sqrt{5}x} = \frac{2}{\sqrt{5}}$

(3) $\tan\theta = \frac{1}{2}$

(4) $\cot\theta = 2$

(5) $\sec\theta = \frac{\sqrt{5}}{2}$

(6) $\operatorname{cosec}\theta = \sqrt{5}$

Que 5:- If cosec $\theta = \sqrt{10}$ find all the values of all T-ratios of θ

As we know that $\operatorname{cosec}\theta = \frac{H}{P}$

$\therefore \operatorname{cosec}\theta = \frac{AC}{AB}$ — (1)

Now,

$\Rightarrow \operatorname{cosec}\theta = \frac{\sqrt{10}\,x}{1x}$ — (2)

$\Rightarrow \frac{AC}{AB} = \frac{\sqrt{10}\,x}{1x}$

By Pythagorous theorem, we know that

$P^2 + B^2 = H^2$

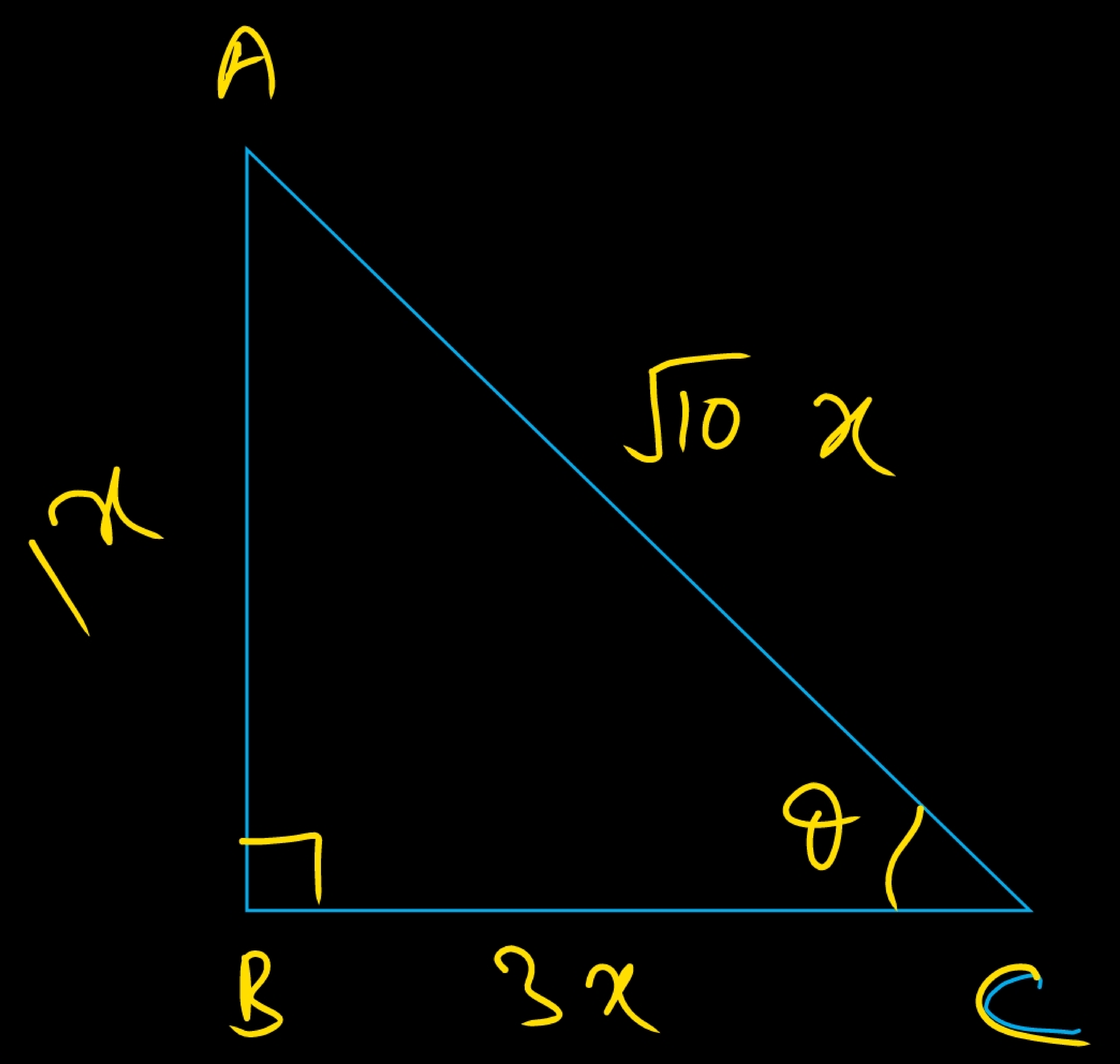
$\Rightarrow (1x)^2 + (BC)^2 = (\sqrt{10}\,x)^2$

$\Rightarrow x^2 + BC^2 = 10x^2$

$\Rightarrow BC^2 = 10x^2 - x^2$

$\Rightarrow BC = \sqrt{9x^2}$

$\Rightarrow \boxed{BC = 3x}$



$\therefore$ (1) $\sin\theta = \frac{1x}{\sqrt{10}\,x} = \frac{1}{\sqrt{10}}$

(2) $\cos\theta = \frac{3x}{\sqrt{10}\,x} = \frac{3}{\sqrt{10}}$

(3) $\tan\theta = \frac{1}{3}$

(4) $\cot\theta = 3$

(5) $\sec\theta = \sqrt{10}/3$

(6) $\operatorname{cosec}\theta = \sqrt{10}$

Que 6:- If $\sin\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$ find all the values of all T-ratios of θ

As we know that $\sin\theta = \dfrac{P}{H}$

$\therefore \sin\theta = \dfrac{AB}{AC}$

But,

$$\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$$

$$\Rightarrow \frac{AB}{AC} = \frac{a^2 - b^2}{a^2 + b^2}$$

By Pythagorous theorem

$$P^2 + B^2 = H^2$$

$AB^2 + BC^2 = AC^2$

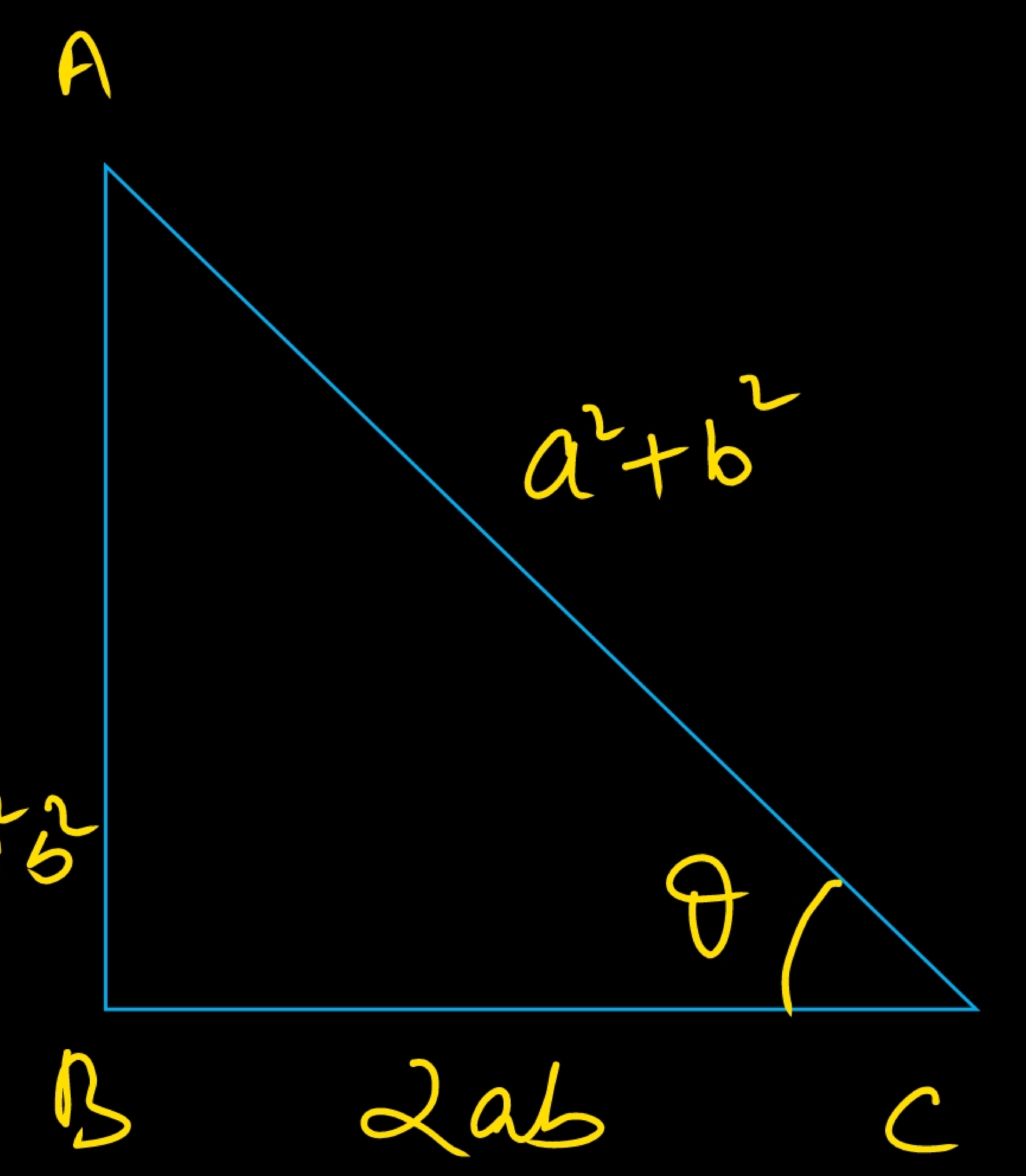
$\Rightarrow (a^2 - b^2)^2 + BC^2 = (a^2 + b^2)^2$

$\Rightarrow \cancel{a^4} + \cancel{b^4} - 2a^2 \cdot b^2 + BC^2 = \cancel{a^4} + \cancel{b^4} + 2a^2b^2$

$\Rightarrow BC^2 = 4a^2b^2$

$\Rightarrow BC = \sqrt{4a^2b^2}$

$\Rightarrow BC = 2ab$



① $\sin\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$

② $\cos\theta = \dfrac{2ab}{a^2 + b^2}$

③ $\tan\theta = \dfrac{a^2 - b^2}{2ab}$

④ $\cot\theta = \dfrac{2ab}{a^2 - b^2}$

⑤ $\sec\theta = \dfrac{a^2 + b^2}{2ab}$

⑥ $\text{Cosec}\,\theta = \dfrac{a^2 + b^2}{a^2 - b^2}$

7:- If $\sin\theta = \dfrac{c}{\sqrt{c^2+d^2}}$, where $d > 0$ then find the values of $\cos\theta$ and $\tan\theta$.

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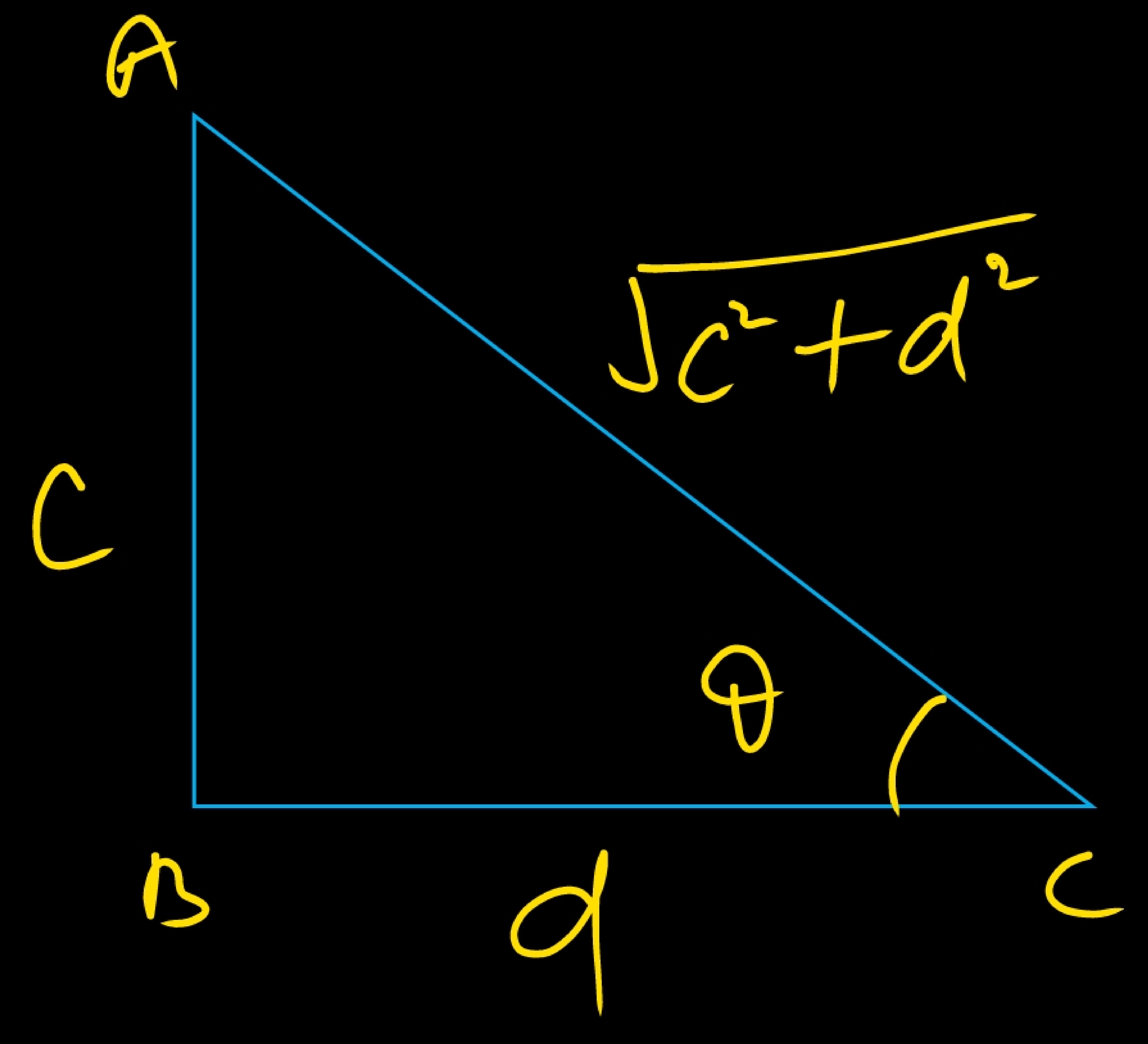


As we know that $\sin\theta = \dfrac{P}{H}$

$\therefore \sin\theta = \dfrac{AB}{AC}$ — ①

$\sin\theta = \dfrac{c}{\sqrt{c^2+d^2}}$ — ②

from eqⁿ ① & ②

$\dfrac{AB}{AC} = \dfrac{c}{\sqrt{c^2+d^2}}$

$\therefore$ By Pythagorous theorem

$P^2 + B^2 = H^2$

$\Rightarrow AC^2 + BC^2 = AC^2$

$\Rightarrow c^2 + BC^2 = \left(\sqrt{c^2+d^2}\right)^2$

$\Rightarrow \cancel{c^2} + BC^2 = \cancel{c^2} + d^2$

$\Rightarrow BC^2 = d^2$

$\Rightarrow \boxed{BC = d}$

① $\sin\theta = \dfrac{c}{\sqrt{c^2+d^2}}$

② $\cos\theta = \dfrac{d}{\sqrt{c^2+d^2}}$

③ $\tan\theta = \dfrac{c}{d}$

④ $\cot\theta = \dfrac{d}{c}$

⑤ $\sec\theta = \dfrac{\sqrt{c^2+d^2}}{d}$

⑥ $\operatorname{cosec}\theta = \dfrac{\sqrt{c^2+d^2}}{c}$

8:- If $\sqrt{3}\tan\theta = 1$ then evaluate $(\cos^2\theta - \sin^2\theta)$.

As we know that $\tan\theta = \frac{P}{B}$

$\therefore \tan\theta = \frac{AB}{BC}$ — (1)

now,

$\sqrt{3}\tan\theta = 1$

$\Rightarrow \tan\theta = \frac{1x}{\sqrt{3}x}$ — (2)

from eqⁿ (1) & (2)

$\frac{AB}{BC} = \frac{x}{\sqrt{3}x}$

By the Pythagarous theorem

$P^2 + B^2 = H^2$

$\Rightarrow AB^2 + BC^2 = AC^2$

$\Rightarrow x^2 + (\sqrt{3}x)^2 = AC^2$

$\Rightarrow 1x^2 + 3x^2 = AC^2$

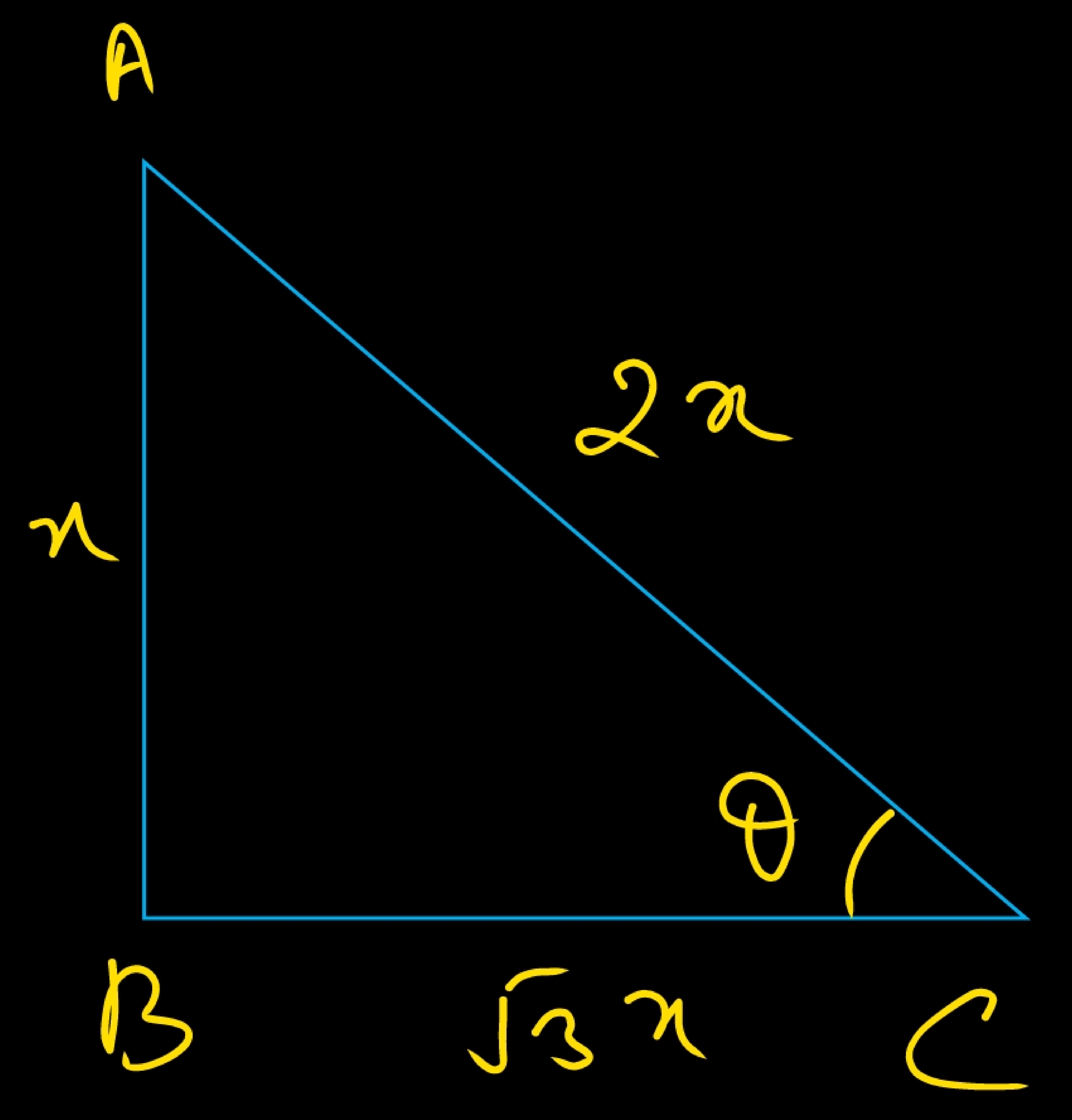
$\Rightarrow 4x^2 = AC^2$

$\Rightarrow \sqrt{4x^2} = AC$

$\Rightarrow \boxed{2x = AC}$

$\therefore \cos^2\theta - \sin^2\theta$

$= \left(\frac{\sqrt{3}\cancel{x}}{2\cancel{x}}\right)^2 - \left(\frac{\cancel{x}}{2\cancel{x}}\right)^2$



$\Rightarrow \frac{3}{4} - \frac{1}{4}$

$\Rightarrow \frac{3-1}{4}$

$\Rightarrow \frac{\cancel{2}}{\cancel{4}}$

$\Rightarrow \frac{1}{2}$ Ans

9:- If $4\tan\theta = 3$ then prove that $\sin\theta\cos\theta = \frac{12}{25}$.

$\Rightarrow 4\tan\theta = 3$ (given)

$\Rightarrow \tan\theta = \frac{3x}{4x}$ — (i)

As we know that $\tan\theta = \frac{P}{B}$

$\therefore \tan\theta = \frac{AB}{BC}$ — (2)

from eqn (1) & (2)

$\frac{AB}{BC} = \frac{3x}{4x}$

By Pythagorous theorem

$P^2 + B^2 = H^2$

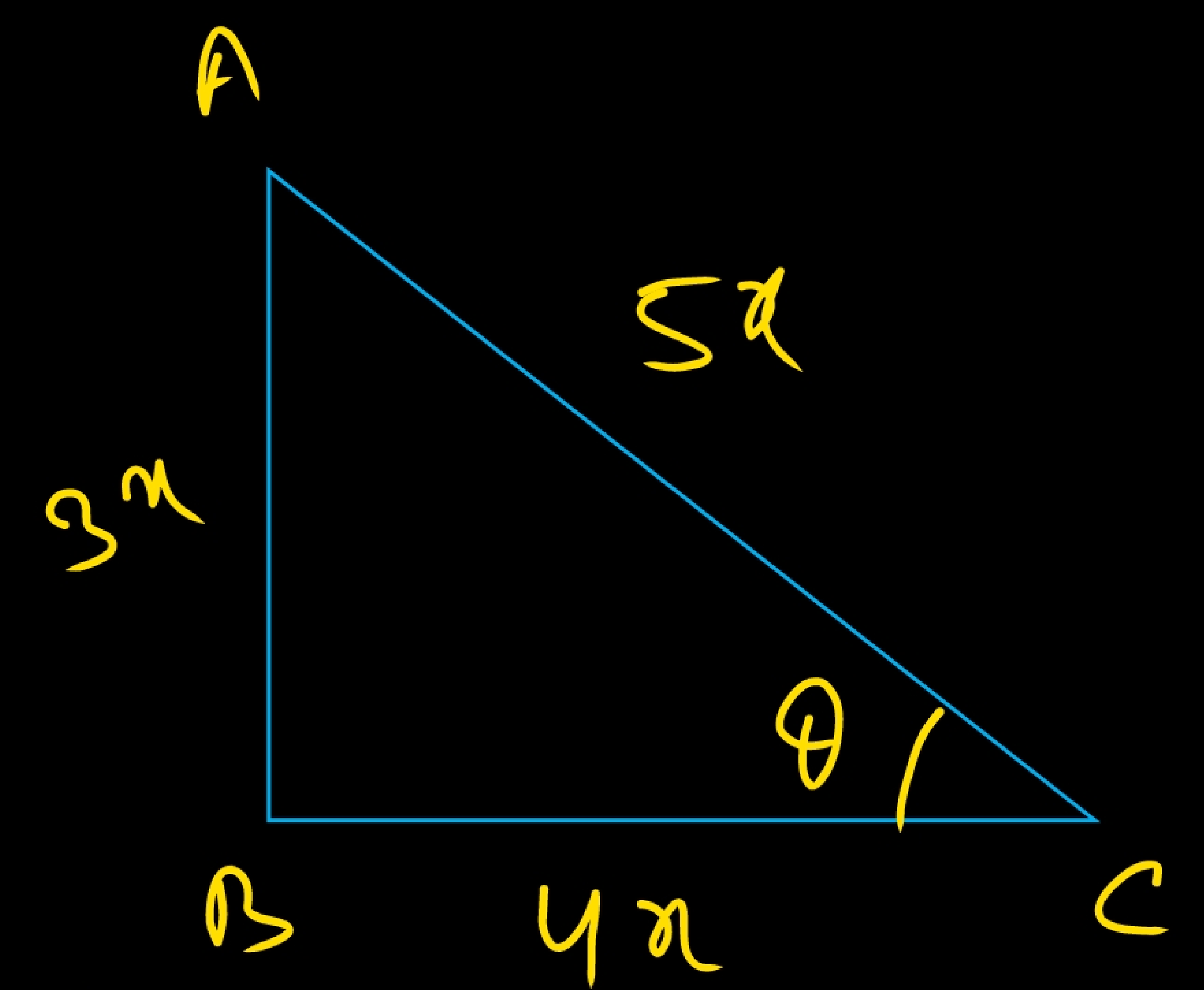
$\Rightarrow (3x)^2 + (4x)^2 = AC^2$

$\Rightarrow 9x^2 + 16x^2 = AC^2$

$\Rightarrow 25x^2 = AC^2$

$\Rightarrow \sqrt{25x^2} = AC$

$\Rightarrow \boxed{AC = 5x}$



L.H.S =

$\therefore \sin\theta \cdot \cos\theta$

$\Rightarrow \frac{3\cancel{x}}{5\cancel{x}} \times \frac{4\cancel{x}}{5\cancel{x}}$

$\Rightarrow \frac{12}{25} = $ R.H.S Proved

10:- If $\sin\theta = \dfrac{a}{b}$ then prove that $(\sec\theta + \tan\theta) = \sqrt{\dfrac{b+a}{b-a}}$.

As we know that $\sin\theta = \dfrac{P}{H}$

$\therefore \sin\theta = \dfrac{AB}{AC}$ — (1)

$\sin\theta = \dfrac{a}{b}$ — (2)

$\Rightarrow \dfrac{AB}{AC} = \dfrac{a}{b}$

By Pythagorous theorem

$P^2 + B^2 = H^2$

$\Rightarrow AB^2 + BC^2 = AC^2$

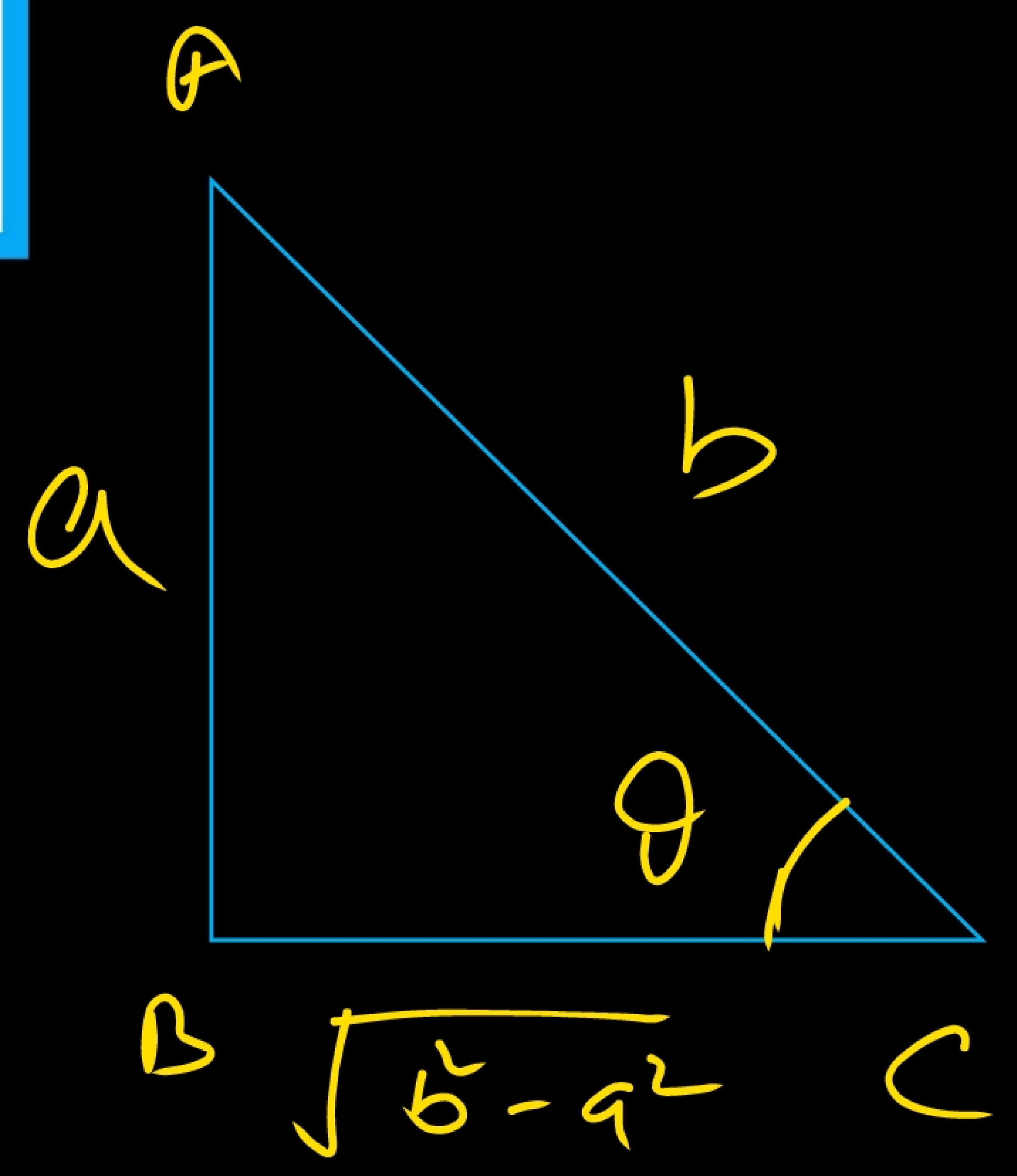
$\Rightarrow a^2 + BC^2 = b^2$

$\Rightarrow BC^2 = b^2 - a^2$

$\Rightarrow \boxed{BC = \sqrt{b^2 - a^2}}$

$\therefore$ L.H.S $= \sec\theta + \tan\theta$

$= \dfrac{b}{\sqrt{b^2-a^2}} + \dfrac{a}{\sqrt{b^2-a^2}}$



$= \dfrac{b+a}{\sqrt{b^2-a^2}}$

$= \dfrac{\sqrt{(b+a)^2}}{\sqrt{(b+a)(b-a)}}$

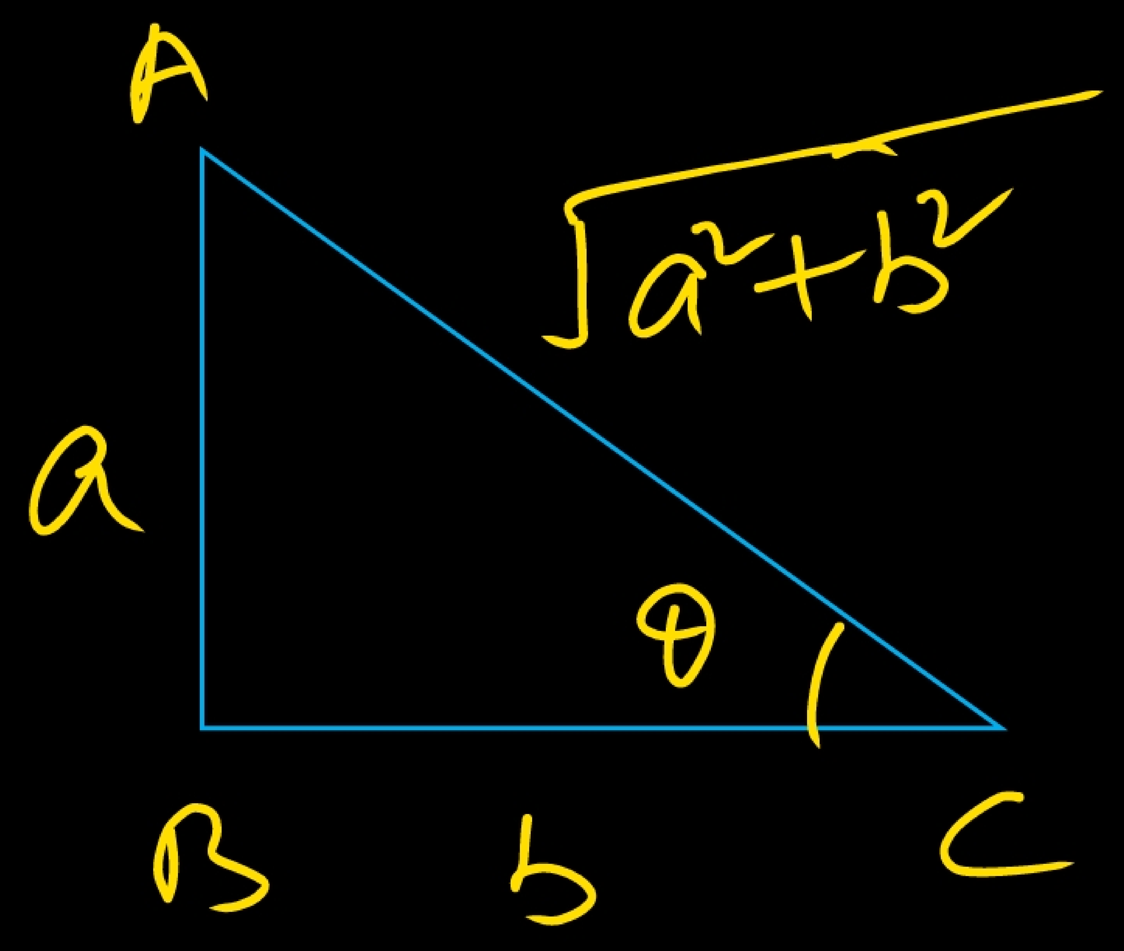
$$= \sqrt{\frac{(b+a)^{\cancel{2}}}{\cancel{(b+a)}(b-a)}}$$

$$\frac{\sqrt{3}}{\sqrt{5}}$$

$$= \sqrt{\frac{b+a}{b-a}} \quad \underline{\underline{\text{Ans}}}$$

$$\sqrt{\frac{3}{5}}$$

Que 11:- If $\tan\theta = \frac{a}{b}$ then prove that $\left(\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}\right) = \frac{(a^2-b^2)}{(a^2+b^2)}$



As we know that $\tan\theta = \frac{P}{B}$

$\therefore \tan\theta = \frac{AB}{BC}$ — (1)

$\tan\theta = \frac{a}{b}$ — (2)

from eqn (1) & (2)

$\frac{AB}{BC} = \frac{a}{b}$

By Pythagoras theorem

$P^2 + B^2 = H^2$

$\Rightarrow AB^2 + BC^2 = AC^2$

$\Rightarrow a^2 + b^2 = AC^2$

$\Rightarrow \sqrt{a^2+b^2} = AC$

L.H.S $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

$= \frac{a \times \frac{a}{\sqrt{a^2+b^2}} - b \times \frac{b}{\sqrt{a^2+b^2}}}{a \times \frac{a}{\sqrt{a^2+b^2}} + b \times \frac{b}{\sqrt{a^2+b^2}}}$

$\Rightarrow \frac{\frac{a^2}{\sqrt{a^2+b^2}} - \frac{b^2}{\sqrt{a^2+b^2}}}{\frac{a^2}{\sqrt{a^2+b^2}} + \frac{b^2}{\sqrt{a^2+b^2}}}$

$\Rightarrow \frac{\frac{a^2-b^2}{\cancel{(\sqrt{a^2+b^2})}}}{\frac{a^2+b^2}{\cancel{(\sqrt{a^2+b^2})}}}$

$$\Rightarrow \frac{a^2-b^2}{a^2+b^2} = R.H.S \quad \text{Proved}$$

Que 12:- If $\sin\theta = \dfrac{12}{13}$ then evaluate $\left(\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}\right)$.

As we know that $\sin\theta = \dfrac{P}{H}$

$\therefore \sin\theta = \dfrac{AB}{AC}$ — (1)

But $\sin\theta = \dfrac{12x}{13x}$ — (2)

from eqn (1) & (2), we get

$$\frac{AB}{AC} = \frac{12x}{13x}$$

By Pythagoras theorem

$$P^2 + B^2 = H^2$$

$\Rightarrow AB^2 + BC^2 = AC^2$

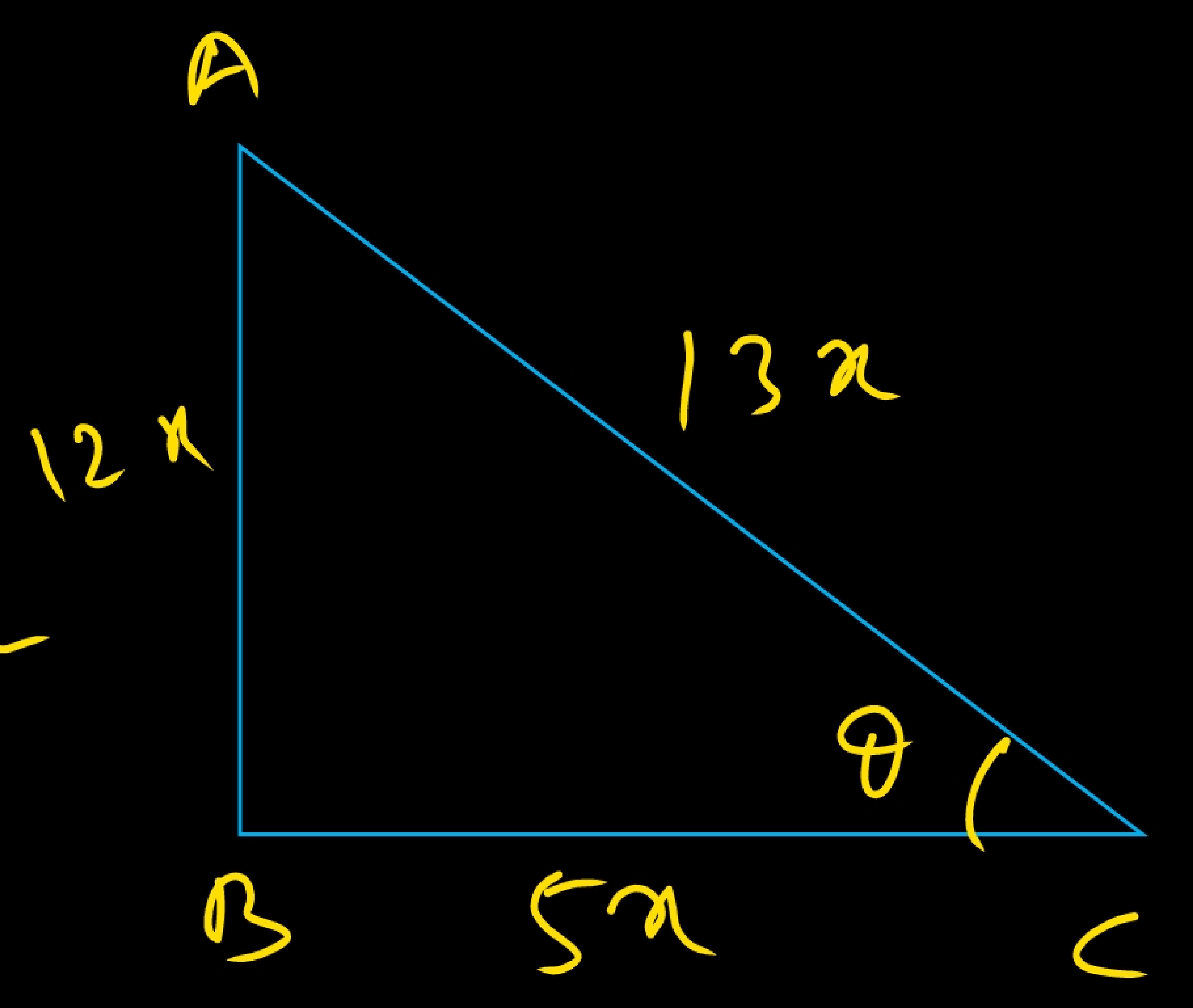
$\Rightarrow (12x)^2 + BC^2 = (13x)^2$

$\Rightarrow 144x^2 + BC^2 = 169x^2$

$\Rightarrow BC^2 = 169x^2 - 144x^2$

$\Rightarrow BC = \sqrt{25x^2}$

$\Rightarrow \boxed{BC = 5x}$



$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$$

$$= \frac{2\times\frac{12\cancel{x}}{13\cancel{x}} - 3\times\frac{5\cancel{x}}{13\cancel{x}}}{4\times\frac{12\cancel{x}}{13\cancel{x}} - 9\times\frac{5\cancel{x}}{13\cancel{x}}}$$

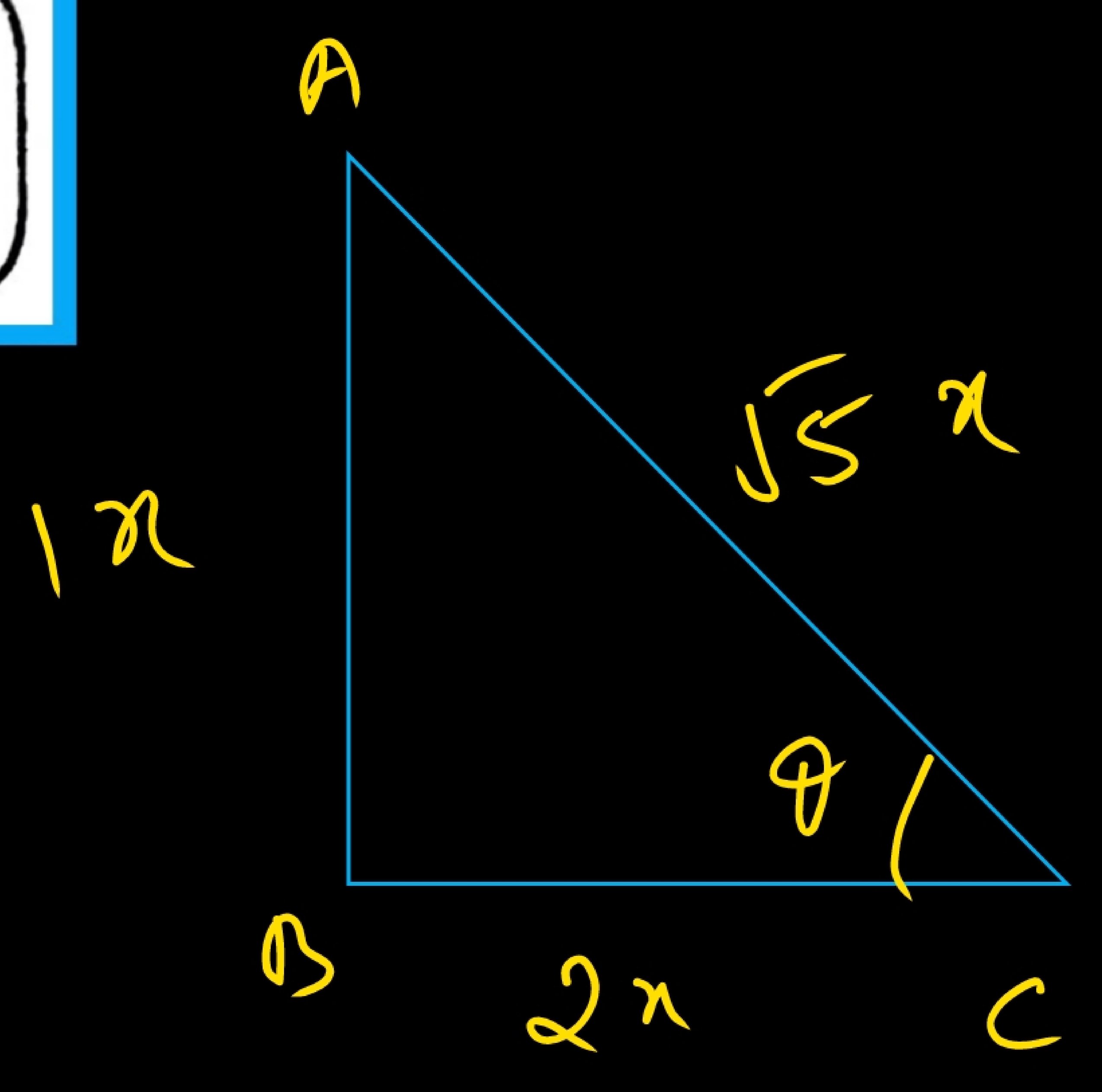
$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$$

$$= \frac{\frac{24-15}{\cancel{13}}}{\frac{48-45}{\cancel{13}}}$$

$$= \frac{\cancel{9}}{3}$$

$$= 3 \text{ Ans}$$

Que 13:- If $\tan\theta = \frac{1}{2}$ then evaluate $\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}\right)$



As we know that $\tan\theta = \frac{P}{B}$

$\therefore \tan\theta = \frac{AB}{BC}$ — ①

But $\tan\theta = \frac{1x}{2x}$ — ②

$\frac{AB}{BC} = \frac{1x}{2x}$

By Pythagorous theorm

$P^2 + B^2 = H^2$

$\Rightarrow AB^2 + BC^2 = AC^2$

$\Rightarrow x^2 + (2x)^2 = AC^2$

$\Rightarrow x^2 + 4x^2 = AC^2$

$\Rightarrow 5x^2 = AC^2$

$\Rightarrow \sqrt{5x^2} = AC$

$\Rightarrow \boxed{AC = \sqrt{5}\,x}$

$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$\Rightarrow \frac{\frac{2\cancel{x}}{\cancel{\sqrt{5}x}}}{\frac{\cancel{x}}{\cancel{\sqrt{5}x}}} + \frac{\frac{\cancel{x}}{\sqrt{5}\cancel{x}}}{\frac{1}{1} + \frac{2\cancel{x}}{\sqrt{5}\cancel{x}}}$

$$= 2 + \frac{\frac{1}{\cancel{\sqrt{5}}}}{\frac{\sqrt{5}+2}{\cancel{\sqrt{5}}}}$$

$$= 2 + \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$= 2 + \frac{\sqrt{5}-2}{\sqrt{5}^2 - 2^2}$$

$$= 2 + \frac{\sqrt{5}-2}{5-4}$$

$$= 2 + \frac{\sqrt{5}-2}{1}$$

$$= \cancel{2} + \sqrt{5} - \cancel{2}$$

$$= \sqrt{5} \text{ Ans}$$

Que 14:- If $\sin\alpha = \frac{1}{2}$ then prove that $(3\cos\alpha - 4\cos^3\alpha) = 0$.

As we know that $\sin\theta = \frac{P}{H}$

$\therefore \sin\alpha = \frac{AB}{AC}$ — (1)

But $\sin\alpha = \frac{1x}{2x}$ — (2)

from eqⁿ (1) & (2)

$\frac{AB}{AC} = \frac{1x}{2x}$

By Pythagorous theorem

$\Rightarrow AB^2 + BC^2 = AC^2$

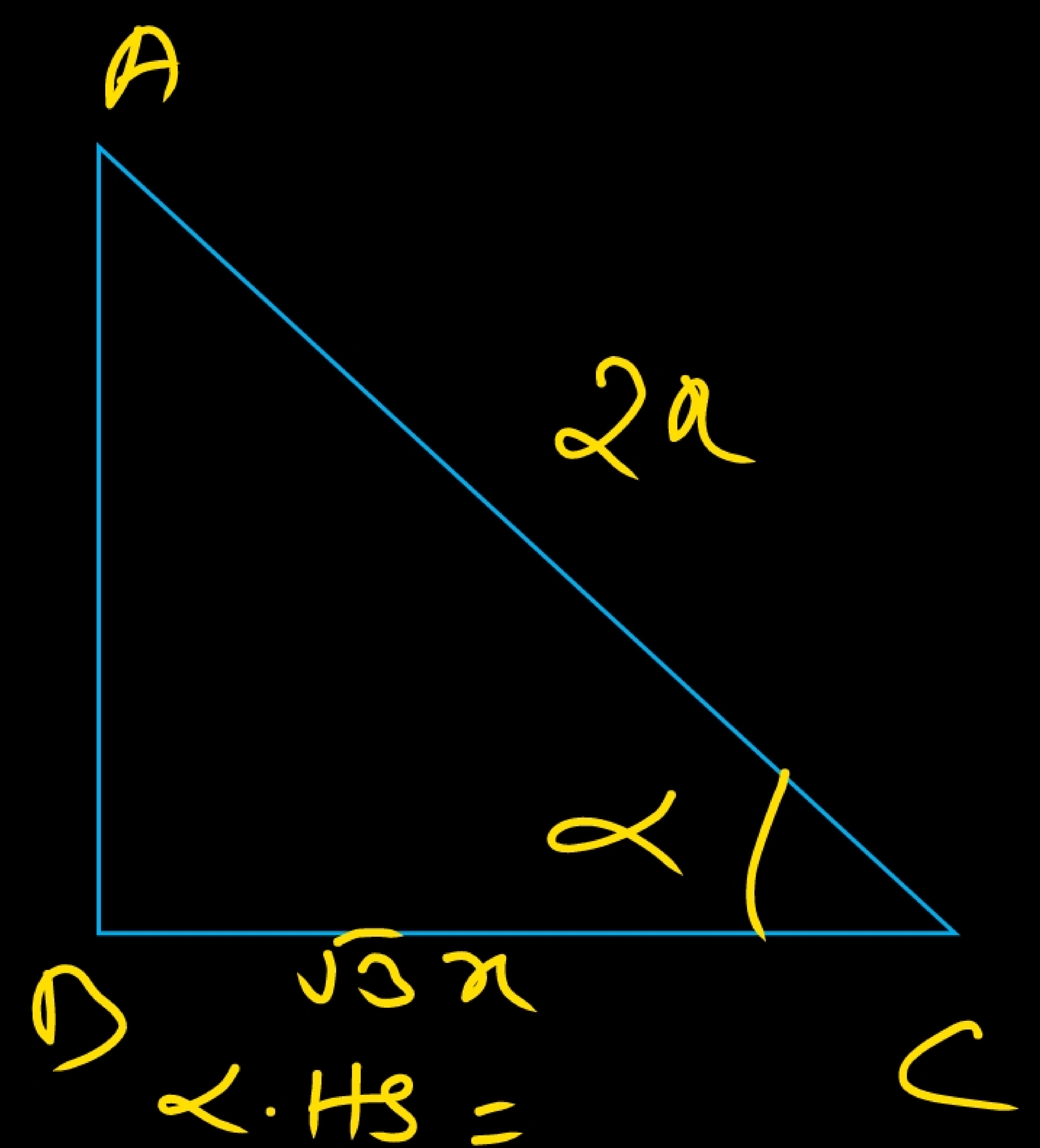
$\Rightarrow x^2 + BC^2 = (2x)^2$

$\Rightarrow x^2 + BC^2 = 4x^2$

$\Rightarrow BC^2 = 4x^2 - x^2$

$\Rightarrow BC = \sqrt{3x^2}$

$\Rightarrow BC = \sqrt{3}\,x$



L.H.S =

$3\cos\alpha - 4\cos^3\alpha$

$3 \times \frac{\sqrt{3}\cancel{x}}{2\cancel{x}} - 4\left(\frac{\sqrt{3}\cancel{x}}{2\cancel{x}}\right)^3$

$= \frac{3\sqrt{3}}{2} - \frac{\cancel{4} \times 3\sqrt{3}}{\cancel{8}\,2}$

$$= \frac{\cancel{3\sqrt{3}} - \cancel{3\sqrt{3}}}{2}$$

$$= \frac{0}{2}$$

$$= 0 = R.HS$$

Proved

Que 15:- If $3\cot\theta = 2$ then prove that $\dfrac{(4\sin\theta - 3\cos\theta)}{(2\sin\theta + 6\cos\theta)} = \dfrac{1}{3}$

$3\cot\theta = 2$ (given)

$\cot\theta = \dfrac{2}{3}$

Now

$$\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta} = \frac{1}{3}$$

$$L.H.S = \frac{4\dfrac{\cancel{\sin\theta}}{\cancel{\sin\theta}} - 3\dfrac{\cos\theta}{\sin\theta}}{2\dfrac{\cancel{\sin\theta}}{\cancel{\sin\theta}} + 6\dfrac{\cos\theta}{\sin\theta}}$$

{On dividing by $\sin\theta$ in N^r and D^r}

$$= \frac{4\times 1 - 3\cot\theta}{2\times 1 + 6\cot\theta}$$

$$= \frac{4 - \cancel{3}\times\dfrac{2}{\cancel{3}}}{2 + \overset{2}{\cancel{6}}\times\dfrac{2}{\cancel{3}}}$$

$$= \frac{4-2}{2+4}$$

$$= \frac{\cancel{2}}{6}$$

$$= \frac{1}{3} = R.H.S$$

Proved

Que 16:- If $\sec\theta = \frac{17}{8}$ then prove that $\frac{3-4\sin^2\theta}{4\cos^2\theta-3} = \frac{3-\tan^2\theta}{1-3\tan^2\theta}$

As we know that $\sec\theta = \frac{H}{B}$

$\therefore \sec\theta = \frac{AC}{BC}$ — (i)

$\sec\theta = \frac{17x}{8x}$ — (ii)

from eqⁿ (1) & (2) we get

$\frac{AC}{BC} = \frac{17x}{8x}$

By Pythagorous theorem,

$P^2 + B^2 = H^2$

$\Rightarrow AB^2 + BC^2 = AC^2$

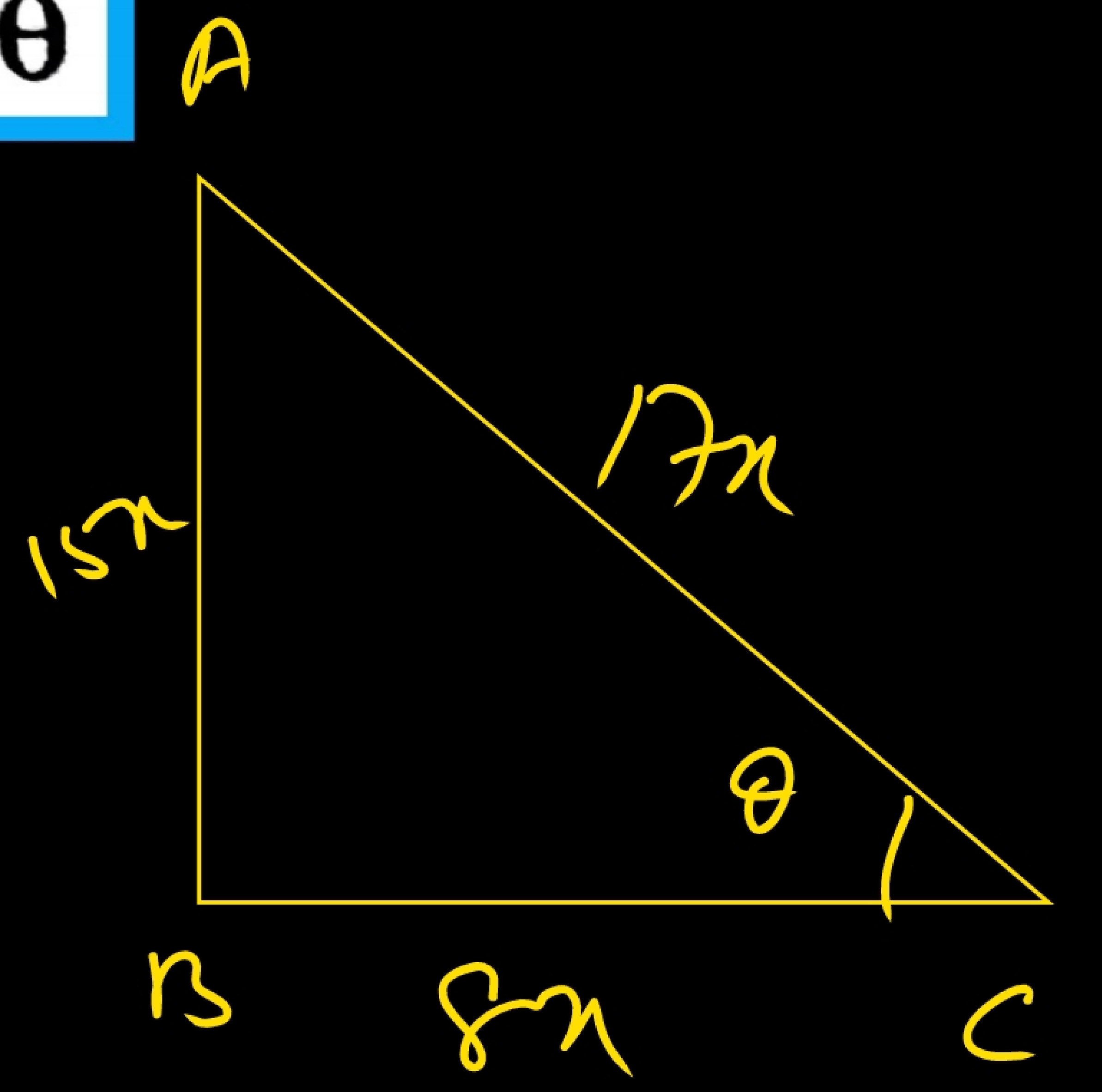
$\Rightarrow AB^2 + (8x)^2 = (17x)^2$

$\Rightarrow AB^2 + 64x^2 = 289x^2$

$\Rightarrow AB^2 = 289x^2 - 64x^2$

$\Rightarrow AB = \sqrt{225x^2}$

$\Rightarrow \boxed{AB = 15x}$



$$\frac{3-4\sin^2\theta}{4\cos^2\theta-3} = \frac{3-\tan^2\theta}{1-3\tan^2\theta}$$

L.HS

$$\frac{3-4\times\left(\frac{15\cancel{x}}{17\cancel{x}}\right)^2}{4\times\left(\frac{8\cancel{x}}{17\cancel{x}}\right)^2-3}$$

$$= \frac{3-4\times\frac{225}{289}}{4\times\frac{64}{289}-\frac{3}{1}}$$

$$\frac{\frac{867-900}{\cancel{289}}}{\frac{256-867}{\cancel{289}}}$$

$$= \frac{\cancel{-}33}{\cancel{-}611}$$

$$= \frac{33}{611}$$

R.HS

$$\frac{3-\tan^2\theta}{1-3\tan^2\theta}$$

$$= \frac{3-\left(\frac{15\cancel{x}}{8\cancel{x}}\right)^2}{1-3\times\left(\frac{15\cancel{x}}{8\cancel{x}}\right)^2}$$

$$= \frac{3-\frac{225}{64}}{1-3\times\frac{225}{64}}$$

$$= \frac{\frac{192-225}{\cancel{64}}}{\frac{64-675}{\cancel{64}}}$$

$$= \frac{192 - 225}{64 - 675}$$

$$= \frac{-33}{-611}$$

$$= \frac{33}{611} = L.H.S = R.H.S$$

(Proved)

Que 17:- If $\tan\theta = \frac{20}{21}$ then prove that $\frac{(1-\sin\theta+\cos\theta)}{(1+\sin\theta+\cos\theta)} = \frac{3}{7}$.

As we know that $\tan\theta = \frac{P}{B}$

$\therefore \tan\theta = \frac{AB}{BC}$ — (1)

But $\tan\theta = \frac{20x}{21x}$ — (2)

$$\frac{AB}{BC} = \frac{20x}{21x}$$

By Pythagorous theorem

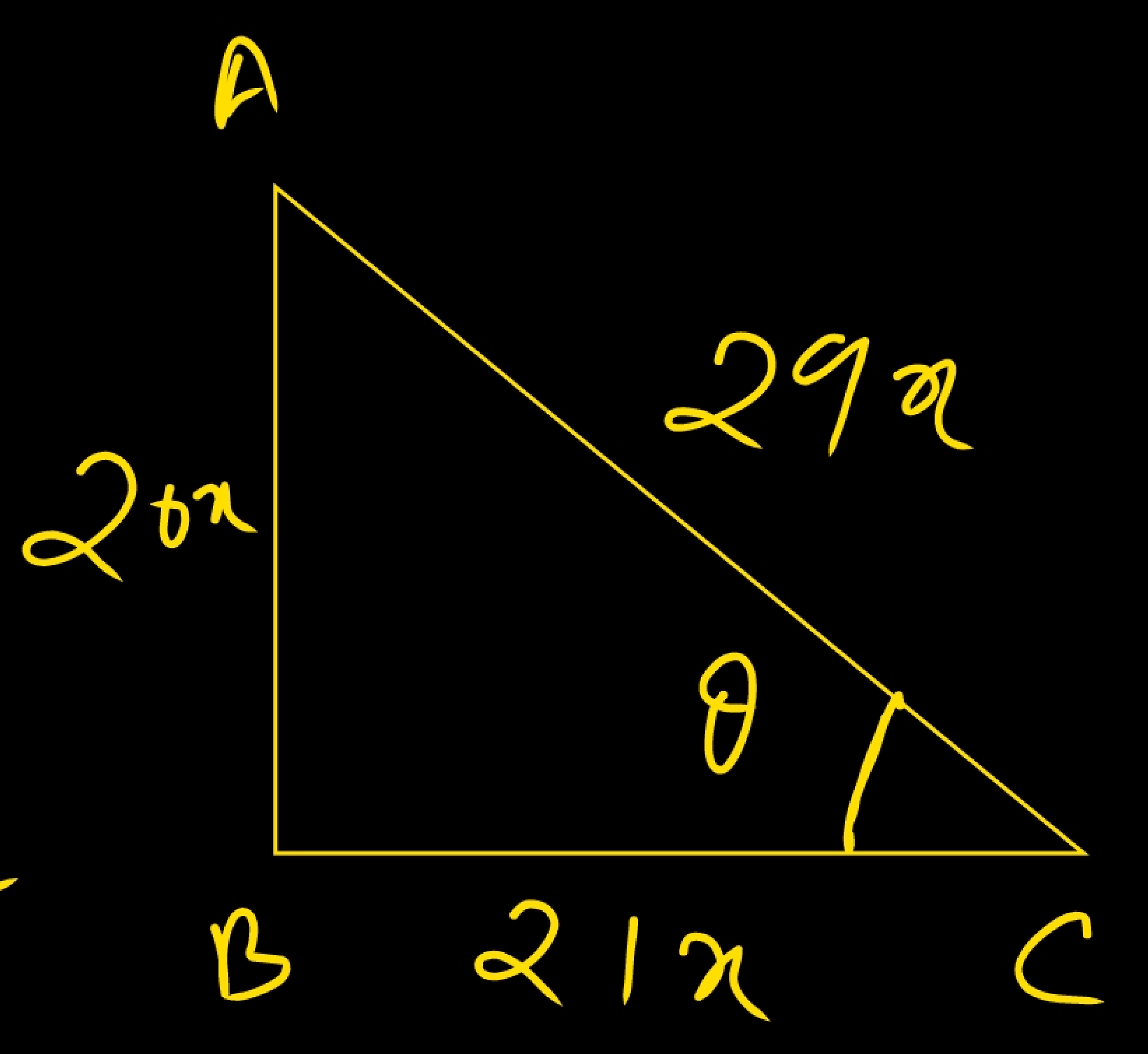
$$P^2 + B^2 = H^2$$

$\Rightarrow (20x)^2 + (21x)^2 = AC^2$

$\Rightarrow 400x^2 + 441x^2 = AC^2$

$\Rightarrow \sqrt{841x^2} = AC$

$\Rightarrow 29x = AC$



$$\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta}=\frac{3}{7}$$

$$L.HS = \frac{1-\frac{20x}{29x}+\frac{21x}{29x}}{1+\frac{20x}{29x}+\frac{21x}{29x}}$$

$$= \frac{\frac{29x-20x+21x}{\cancel{29x}}}{\frac{29x+20x+21x}{\cancel{29x}}}$$

$$= \frac{50x-20x}{70x}$$

$$= \frac{3\cancel{0}\cancel{x}}{7\cancel{0}\cancel{x}}$$

$$= \frac{3}{7} = R.HS$$

Que 18:- If $\tan\theta = \frac{1}{\sqrt{7}}$ then prove that $\left(\frac{\operatorname{cosec}^2\theta + \sec^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta}\right) = \frac{4}{3}$.

$\boxed{\tan\theta = \frac{1}{\sqrt{7}}}$ $\quad \therefore \boxed{\cot\theta = \frac{\sqrt{7}}{1}}$

formula

$\sec^2\theta - \tan^2\theta = 1$

$\boxed{\sec^2\theta = 1 + \tan^2\theta}$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\boxed{\operatorname{cosec}^2\theta = 1 + \cot^2\theta}$

L.HS $\quad \frac{\operatorname{cosec}^2\theta + \sec^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta}$

$$= \frac{1 + \cot^2\theta + 1 + \tan^2\theta}{1 + \cot^2\theta - (1 + \tan^2\theta)}$$

$$= \frac{2 + (\sqrt{7})^2 + \left(\frac{1}{\sqrt{7}}\right)^2}{\cancel{1} + (\sqrt{7})^2 - \cancel{1} - \tan^2\theta}$$

$$= \frac{2 + 7 + \frac{1}{7}}{7 - \frac{1}{7}}$$

$$= \frac{9 + \frac{1}{7}}{7 - \frac{1}{7}}$$

$$= \frac{\frac{63+1}{\cancel{7}}}{\frac{49-1}{\cancel{7}}}$$

$$= \frac{64}{48} = \frac{\cancel{8}^{4}}{\cancel{6}_{3}}$$

$$= \frac{4}{3} = R.H.S$$

Que 19:- If $\sin\theta = \frac{3}{4}$ then prove that $\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$.

As. we know that $\sin\theta = \frac{P}{H}$

$\therefore \sin\theta = \frac{AB}{AC}$ — (1)

But $\sin\theta = \frac{3x}{4x}$ — (2)

from eq^n (1) & (2)

$\frac{AB}{AC} = \frac{3x}{4x}$

By Pythagoras theorem

$P^2 + B^2 = H^2$

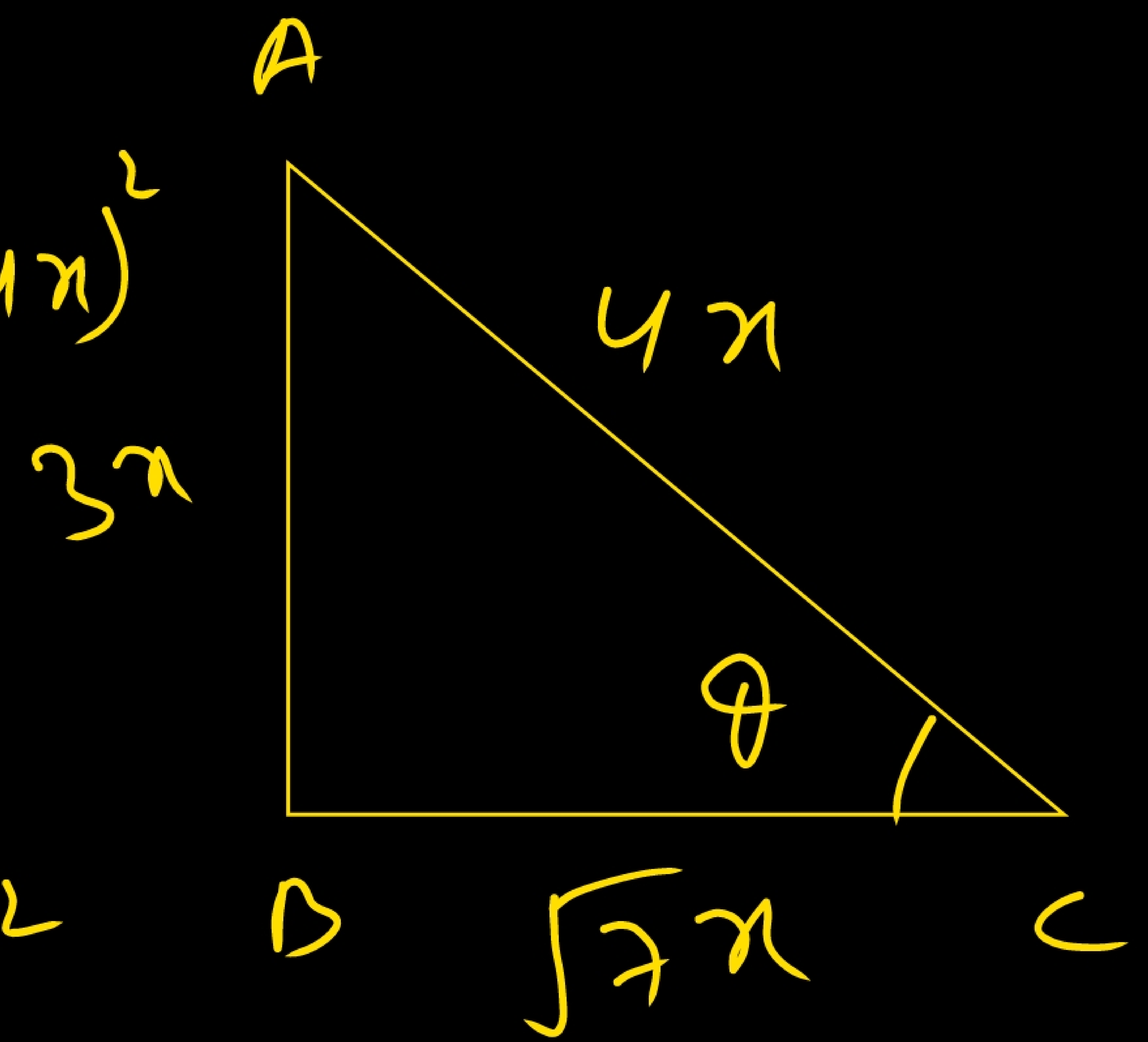
$\Rightarrow (3x)^2 + BC^2 = (4x)^2$

$\Rightarrow 9x^2 + BC^2 = 16x^2$

$\Rightarrow BC^2 = 16x^2 - 9x^2$

$\Rightarrow BC = \sqrt{7x^2}$

$\Rightarrow BC = \sqrt{7}\,x$



$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$$

$$= \sqrt{\frac{1}{\tan^2\theta}}$$

$$\left\{ \because \operatorname{cosec}^2\theta - \cot^2\theta = 1 \quad \& \quad \sec^2\theta - 1 = \tan^2\theta \right\}$$

$$= \sqrt{\cot^2\theta}$$

$$= \cot\theta$$

$$\therefore = \frac{\sqrt{7}\cancel{x}}{3\cancel{x}}$$

$$= \frac{\sqrt{7}}{3} = \text{R.H.S. Proved}$$

Que 20:- If $3\tan A = 4$ then prove that

(i) $\sqrt{\dfrac{\sec A - \operatorname{cosec} A}{\sec A + \operatorname{cosec} A}} = \dfrac{1}{\sqrt{7}}$ (ii) $\sqrt{\dfrac{1-\sin A}{1+\cos A}} = \dfrac{1}{2\sqrt{2}}$

As we know that $\tan\theta = \dfrac{P}{B}$

$\therefore \tan\theta = \dfrac{BC}{AB}$ — (1)

But $\tan A = \dfrac{4x}{3x}$ — (2)

from eqⁿ (1) & (2)

$\dfrac{BC}{AB} = \dfrac{4x}{3x}$

By Pythagoras theorem

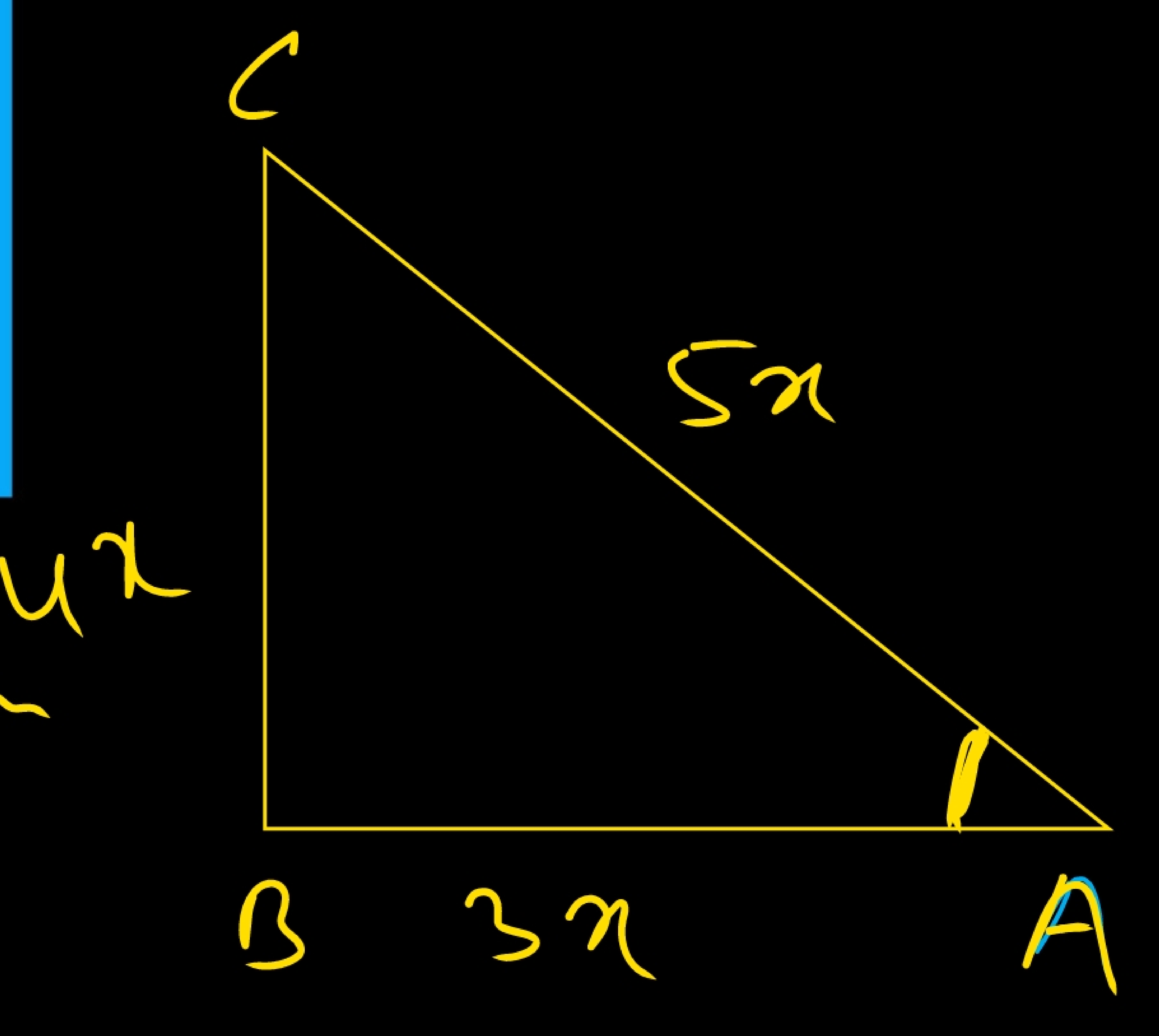
$P^2 + B^2 = H^2$

$\Rightarrow (BC)^2 + (AB)^2 = AC^2$

$\Rightarrow (4x)^2 + (3x)^2 = AC^2$

$\Rightarrow 16x^2 + 9x^2 = AC^2$

$\Rightarrow \sqrt{25x^2} = AC$



$\therefore \boxed{AC = 5x}$

① $\sqrt{\dfrac{\sec A - \operatorname{cosec} A}{\sec A + \operatorname{cosec} A}} = \dfrac{1}{\sqrt{7}}$

L.H.S

$$\sqrt{\dfrac{\dfrac{5\cancel{x}}{3\cancel{x}} - \dfrac{5\cancel{x}}{4\cancel{x}}}{\dfrac{5\cancel{x}}{3\cancel{x}} + \dfrac{5\cancel{x}}{4\cancel{x}}}}$$

$$= \sqrt{\dfrac{\dfrac{20-15}{\cancel{12}}}{\dfrac{20+15}{\cancel{12}}}}$$

$$= \sqrt{\dfrac{\cancel{5}}{\cancel{35}}}$$

$$= \sqrt{\dfrac{1}{7}}$$

$$= \dfrac{1}{\sqrt{7}} = R.H.S$$

(11) $$\sqrt{\frac{1-\sin A}{1+\cos A}} = \frac{1}{2\sqrt{2}}$$

$$= \sqrt{\frac{1 - \frac{4\cancel{x}}{5\cancel{x}}}{1 + \frac{3\cancel{x}}{5\cancel{x}}}}$$

$$= \sqrt{\frac{\frac{5-4}{5}}{\frac{5+3}{5}}}$$

$$= \sqrt{\frac{\frac{1}{\cancel{5}}}{\frac{8}{\cancel{5}}}}$$

$$= \sqrt{\frac{1}{8}}$$

$$= \frac{1}{\sqrt{8}}$$

$$= \frac{1}{2\sqrt{2}} = \text{R.H.S Proved}$$

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

Que 21:- If $\cot\theta = \dfrac{15}{8}$ then evaluate $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

Solution:- $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1^2-\sin^2\theta}{1^2-\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\sin^2\theta}$

$\Rightarrow \cot^2\theta$

$\Rightarrow \left(\dfrac{15}{8}\right)^2$

$\Rightarrow \dfrac{225}{64}$ Ans

22:- In a ΔABC, if $\angle B = 90°$ and $\tan A = 1$ then prove that $2\sin A \cos A = 1$.

$\tan A = \frac{1}{1}$

As we know that $\tan\theta = \frac{P}{B}$

$\therefore \tan A = \frac{BC}{AB}$ — ①

Now

$\tan A = \frac{1x}{1x}$ — ②

from eqⁿ ① & ②

$\tan A = \frac{1x}{1x}$

By Pythagorous theorem

$P^2 + B^2 = H^2$

$\Rightarrow (1x)^2 + (1x)^2 = AC^2$

$\Rightarrow AC^2 = 1x^2 + 1x^2$

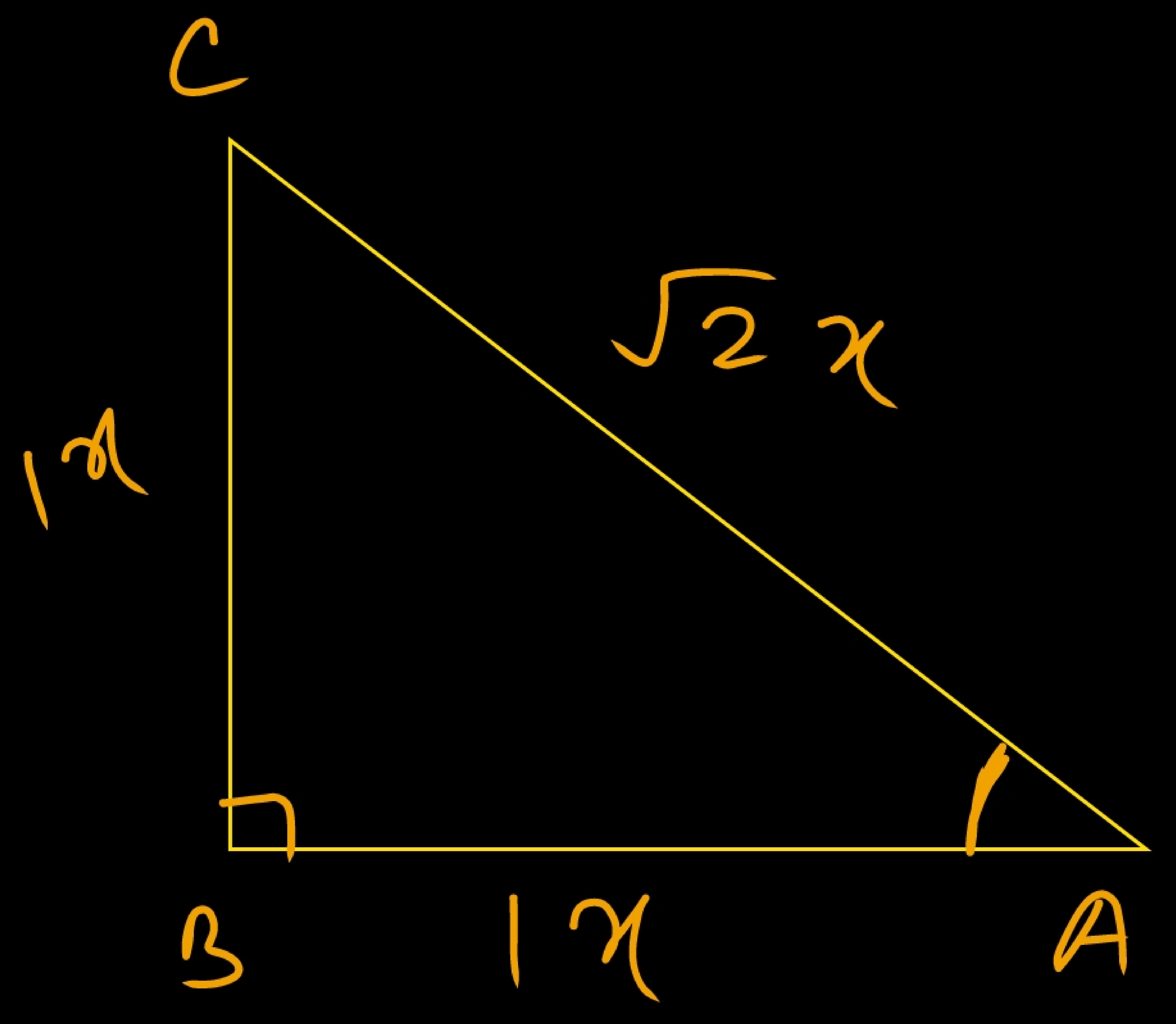
$\Rightarrow AC^2 = 2x^2$

$\Rightarrow \boxed{AC = \sqrt{2}\,x}$

L.HS $= 2 \sin A \cdot \cos A$

$= \cancel{2} \times \frac{1\cancel{x}}{\cancel{\sqrt{2}x}} \times \frac{1\cancel{x}}{\cancel{\sqrt{2}x}}$

$= 1$



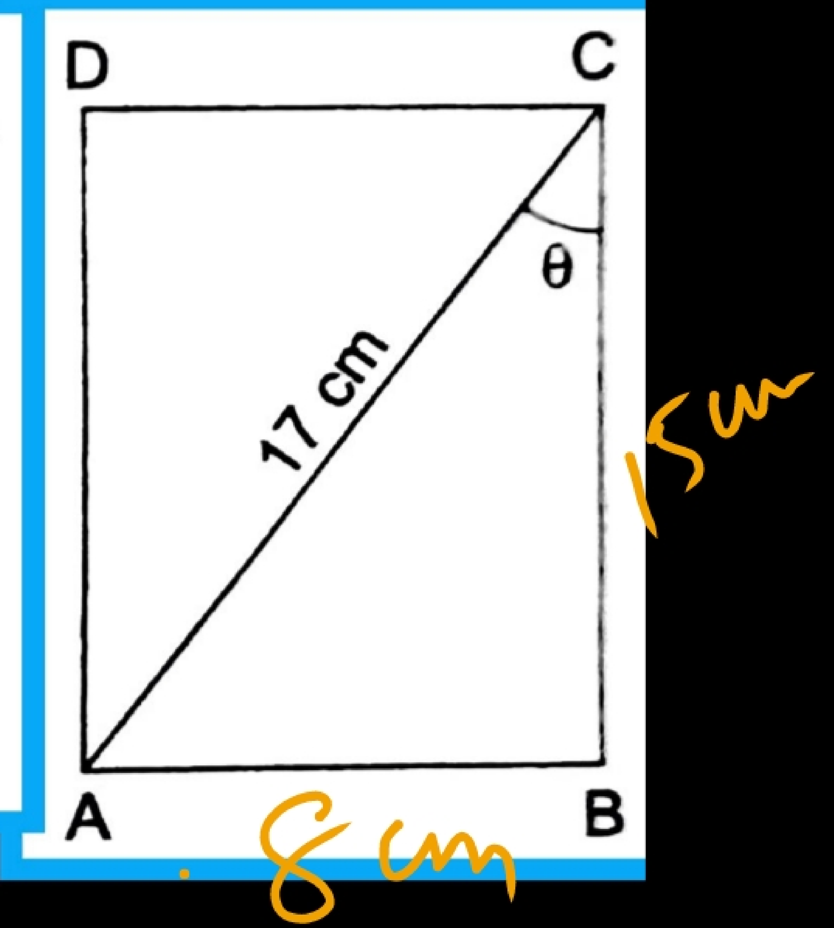
$\therefore$ L.HS = R.HS

Proved

23:- In the given figure, $ABCD$ is a rectangle in which diag. $AC = 17$ cm, $\angle BCA = \theta$ and $\sin\theta = \dfrac{8}{17}$.

Find (i) the area of rect. $ABCD$,

(ii) the perimeter of rect. $ABCD$. [CBSE 2014]



$\sin\theta = \dfrac{AB}{AC}$ — (1)

$\sin\theta = \dfrac{8}{17}$ — (2)

$\therefore \dfrac{AB}{AC} = \dfrac{8}{17}$

By Pythagorous theorem

$P^2 + B^2 = H^2$

$\Rightarrow 8^2 + BC^2 = 17^2$

$\Rightarrow BC = \sqrt{289 - 64}$

$\Rightarrow BC = \sqrt{225}$

$\Rightarrow BC = 15$

$\therefore$ (1) Ar. of rect. $ABCD = l \times B$

$= (8 \times 15)\,cm^2$

$= 120\,cm^2$

(ii) The Perimeter of rect.

$= 2(L + B)$

$= 2(8 + 15)$

$= 2 \times 23$

$= 46$ cm

24:- If $x = \text{cosec } A + \cos A$ and $y = \text{cosec } A - \cos A$ then prove that

$$\left(\frac{2}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1.$$

$x = \text{cosec} A + \cos A$

$y = \text{cosec} A - \cos A$

$\Rightarrow \boxed{x + y = 2 \text{ cosec} A}$

Now, $x - y = \text{cosec} A + \cos A - (\text{cosec} A - \cos A)$

$x - y = \cancel{\text{cosec}} A + \cos A - \cancel{\text{cosec}} A + \cos A$

$\boxed{x - y = 2 \cos A}$

$L.HS = \left(\frac{2}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2$

$= \left(\frac{\cancel{2}}{\cancel{2} \text{ cosec} A}\right)^2 + \left(\frac{\cancel{2} \cos A}{\cancel{2}}\right)^2$

$= (\sin A)^2 + (\cos A)^2$

$= \sin^2 A + \cos^2 A$

$= 1.$ R.HS

25:- If $x = \cot A + \cos A$ and $y = \cot A - \cos A$ then prove that

$$\left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1.$$

$x = \cot A + \cos A \quad —(1)$

$y = \cot A - \cos A \quad —(2)$

$$\boxed{x+y = 2\cot A}$$

$x - y = \cot A + \cos A - (\cot A - \cos A)$

$= \cancel{\cot A} + \cos A - \cancel{\cot A} + \cos A$

$x - y = 2\cos A$

$\underline{L.H.S} = \left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2$

$= \left(\frac{\cancel{2}\cos A}{\cancel{2}\cot A}\right)^2 + \left(\frac{\cancel{2}\cos A}{\cancel{2}}\right)^2$

$= \left(\frac{\cos A}{\frac{\cos A}{\sin A}}\right)^2 + \cos^2 A$

$= \left(\cancel{\cos A} \times \frac{\sin A}{\cancel{\cos A}}\right)^2 + \cos^2 A$

$= \sin^2 A + \cos^2 A$

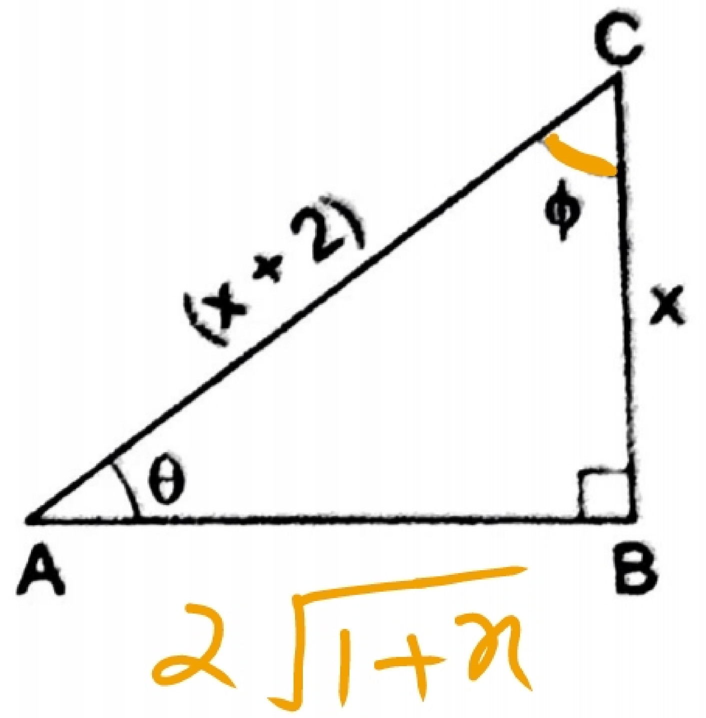
$= 1$

26:- In $\triangle ABC$, $\angle A = \theta$, $\angle B = 90^\circ$ and $\angle C = \phi$. If $BC = x$ and $AC = (x+2)$, find the values of

(i) $(\sqrt{x+1})\cot\phi$,

(ii) $(\sqrt{x^3+x^2})\tan\theta$,

(iii) $\cos\theta$.



Solution:- By Pythagorous theorem

$$\Rightarrow AB^2 + BC^2 = AC^2$$

$$\Rightarrow AB^2 + x^2 = (x+2)^2$$

$$\Rightarrow AB^2 = \cancel{x^2} + 4 + 4x - \cancel{x^2}$$

$$\Rightarrow AB = \sqrt{4(1+x)}$$

$$\Rightarrow AB = 2\sqrt{1+x}$$

(i) $\sqrt{x+1} \times \cot\phi$

$$\therefore \cancel{\sqrt{x+1}} \times \frac{x}{2\cancel{\sqrt{1+x}}}$$

$$\therefore \frac{x}{2} \text{ Ans}$$

(ii) $\sqrt{x^3+x^2}\ \tan\theta$

$$\therefore \sqrt{x^3+x^2} \times \frac{x}{2\sqrt{1+x}}$$

$$= \sqrt{x^2(x+1)} \times \frac{x}{2\sqrt{1+x}}$$

$$= x\cancel{\sqrt{x+1}} \times \frac{x}{2\cancel{\sqrt{1+x}}}$$

$$= \frac{x^2}{2}$$

(iii) $\cos\theta = \dfrac{2\sqrt{1+x}}{x+2}$

27:- If $\cot A + \dfrac{1}{\cot A} = 2$, find the value of $\left(\cot^2 A + \dfrac{1}{\cot^2 A}\right)$

$$\cot A + \frac{1}{\cot A} = 2$$

on Sq. both side

$$\Rightarrow \left(\cot A + \frac{1}{\cot A}\right)^2 = 4$$

$$\Rightarrow \cot^2 A + \frac{1}{\cot^2 A} + 2 \times \cancel{\cot A} \times \frac{1}{\cancel{\cot A}} = 4$$

$$\Rightarrow \boxed{\cot^2 A + \frac{1}{\cot^2 A} = 2}$$

28:- If $\sqrt{3}\tan\theta = 3\sin\theta$, find the value of $\sin\theta$.

$$\sqrt{3}\tan\theta = 3\sin\theta$$

$$\Rightarrow \sqrt{3} \times \frac{\cancel{\sin\theta}}{\cos\theta} = 3\cancel{\sin\theta}$$

$$\Rightarrow \sqrt{3} = 3\cos\theta$$

$$\Rightarrow \frac{\sqrt{3}}{3} = \cos\theta$$

$$\Rightarrow \frac{\cancel{\sqrt{3}}}{\sqrt{3}\times\cancel{\sqrt{3}}} = \cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

As we know that

$$\Rightarrow \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \sin^2\theta + \left(\frac{1}{\sqrt{3}}\right)^2 = 1$$

$$\Rightarrow \sin^2\theta = 1 - \frac{1}{3}$$

$$\Rightarrow \sin^2\theta = \frac{3-1}{3}$$

$$\Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$$

$$\therefore \sin\theta = \frac{\sqrt{2}}{\sqrt{3}} =$$

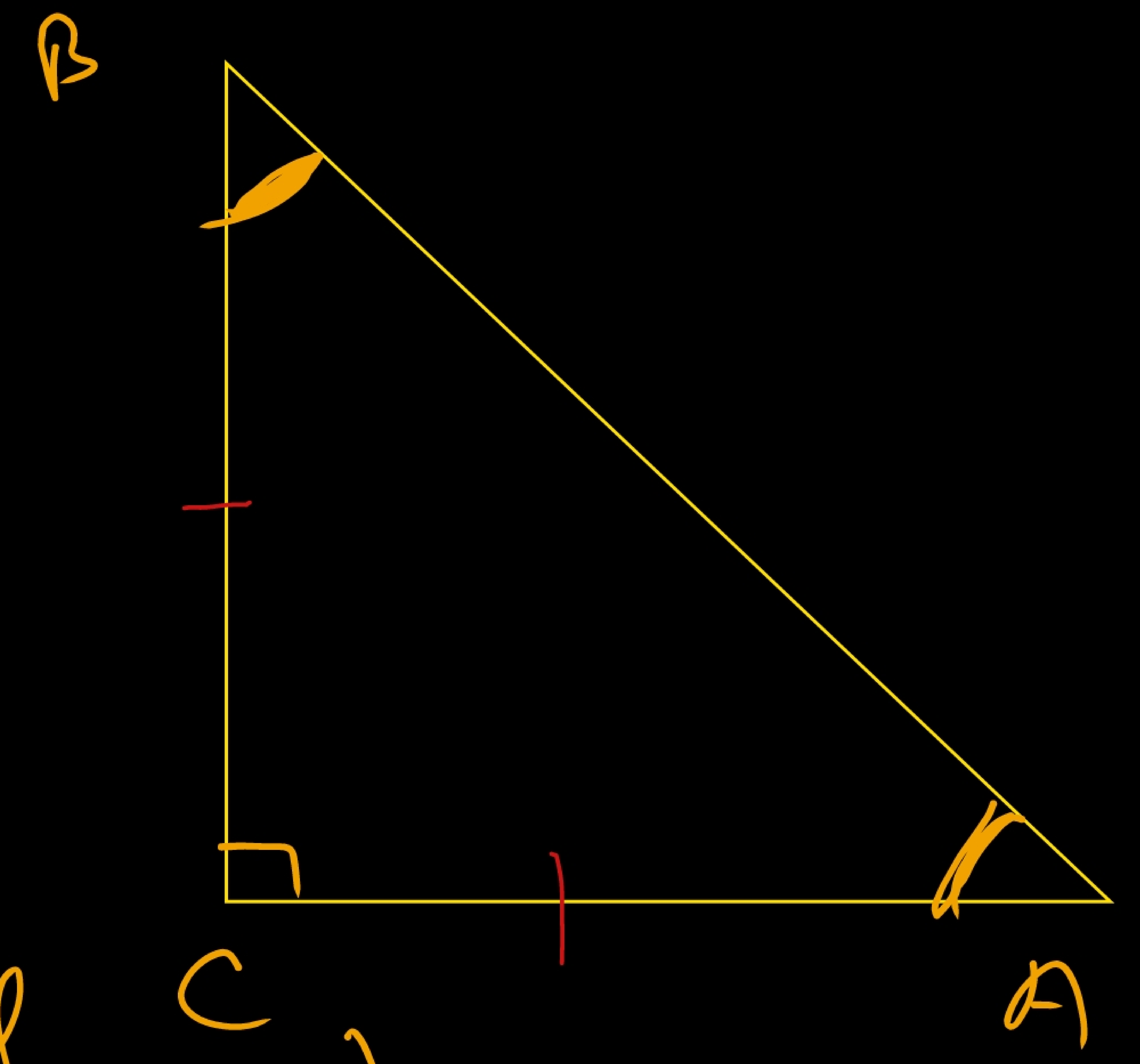
29:- If $\angle A$ and $\angle B$ are acute angles such that $\sin A = \sin B$ then prove that $\angle A = \angle B$.

$$\sin A = \sin B$$

$$\frac{BC}{\cancel{AB}} = \frac{AC}{\cancel{AB}}$$

$$\therefore \quad BC = AC$$

$\therefore \quad \angle A = \angle B$ {Angles opposite to equal sides of a triangle are equal.}



30:- If $\angle A$ and $\angle B$ are acute angles such that $\tan A = \tan B$ then prove that $\angle A = \angle B$.

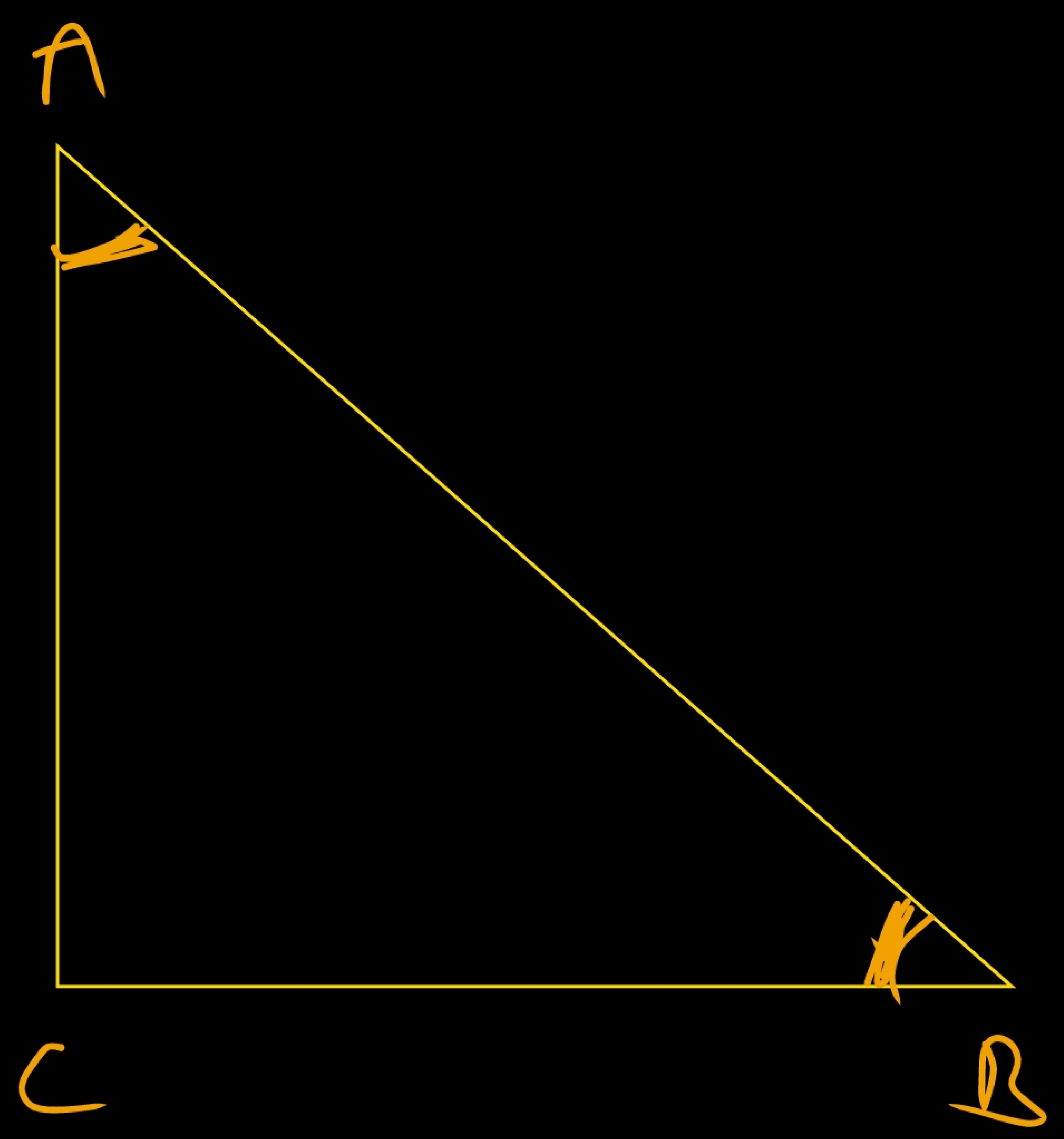
$\tan A = \tan B$

$\Rightarrow \dfrac{BC}{AC} = \dfrac{AC}{BC}$

$\Rightarrow BC^2 = AC^2$

$\therefore \boxed{BC = AC}$

$\therefore \angle A = \angle B$ {Angles opposite to the equal sides of a Δ are equal}



31:- If $1+\sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\frac{1}{2}$.

$\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta = 1+\tan^2\theta$

$\Rightarrow 1+\sin^2\theta = 3\sin\theta\cdot\cos\theta$

On dividing by $\cos^2\theta$ both side.

$\Rightarrow \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cdot\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta}$

$\Rightarrow \sec^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow 1+\tan^2\theta + \tan^2\theta - 3\tan\theta = 0$

$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow 2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$

$\Rightarrow 2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$\Rightarrow (\tan\theta - 1)(2\tan\theta - 1) = 0$

$\therefore \tan\theta - 1 = 0$ or $2\tan - 1 = 0$

$\therefore \tan\theta = 1$ or $\tan\theta = \frac{1}{2}$