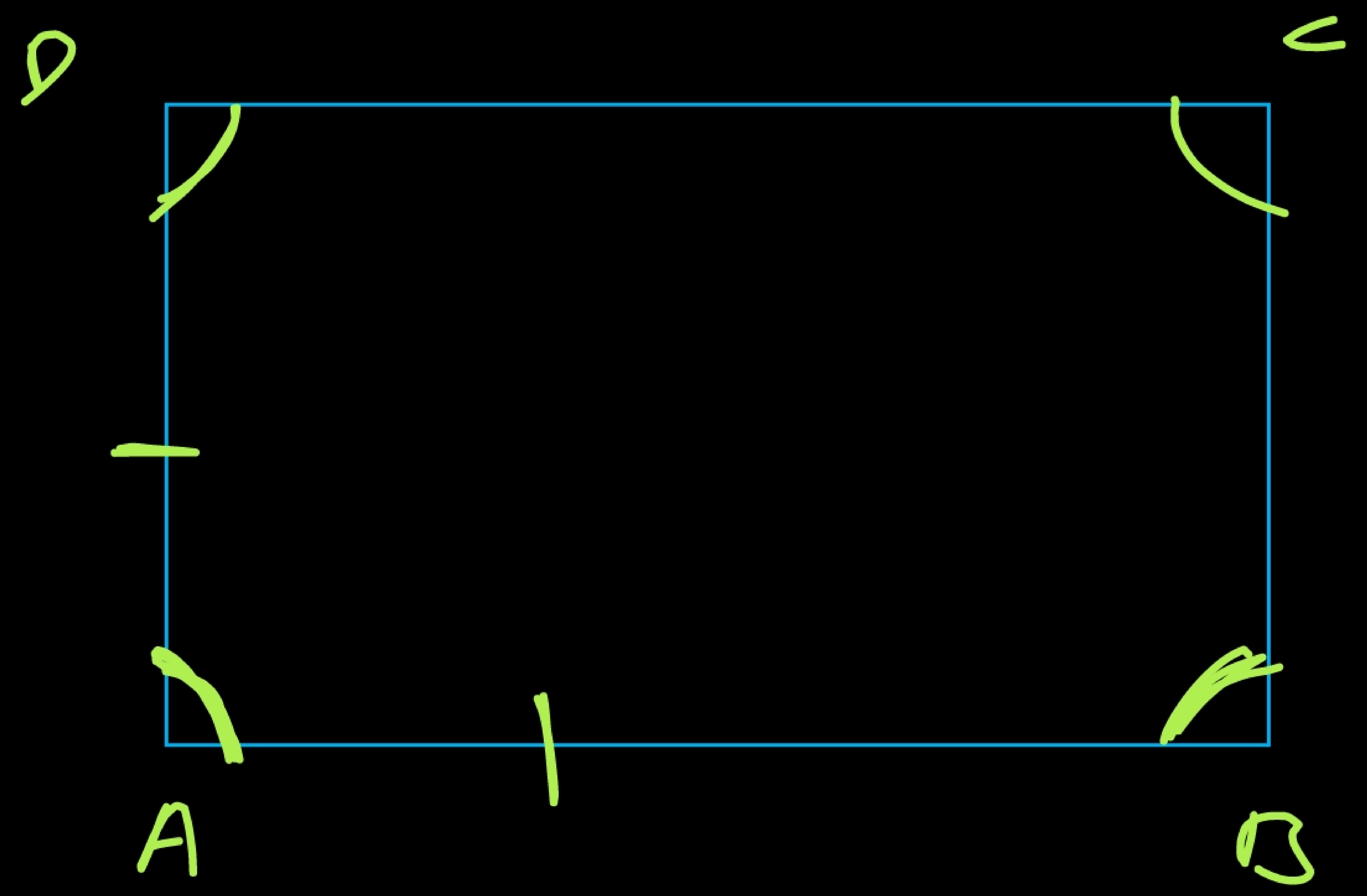
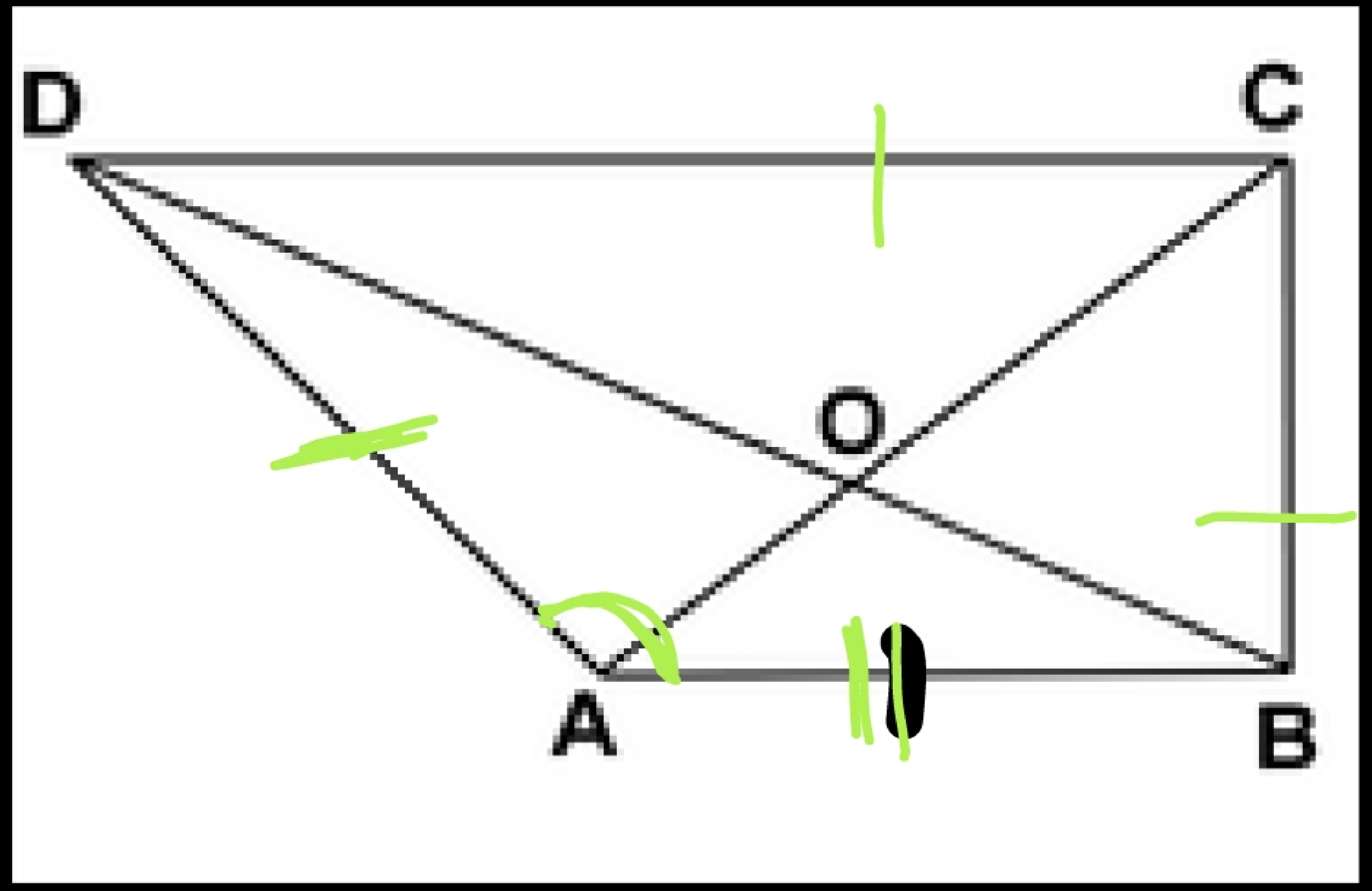

QUADRILATERALS
INTRO DUCTION
Class 9th
iOTA Classes

QUADRILATERAL

A plane figure bounded by four line segments AB, BC, CD and DA is called a quadrilateral, written as quad. ABCD or □ABCD.





(i) VERTICES The points A, B, C, D are called the vertices of quad. ABCD.

(ii) SIDES The line segments AB, BC, CD and DA are called the sides of quad. ABCD.

(iii) DIAGONALS The line segments AC and BD are called the diagonals of quad. ABCD.

(iv) ADJACENT SIDES Two sides of a quadrilateral having a common end point are called its consecutive or adjacent sides.

(AB, BC), (BC, CD), (CD, DA) and (DA, AB) are four pairs of adjacent sides of quad. ABCD.

(V) OPPOSITE SIDES TwO sides of a quadrilateral having no common end point are called its opposite sides. (AB, CD) and (AD, BC) are two pairs of opposite sides of quad. ABCD.

(vi) CONSECUTIVE ANGLES Two angles of a quadrilateral having a common arm are called its consecutive angles.

$\angle A, \angle B$, $\angle C, \angle D$ and $\angle D$, $\angle A$ are four pairs of consecutive angles of a quad. ABCD.

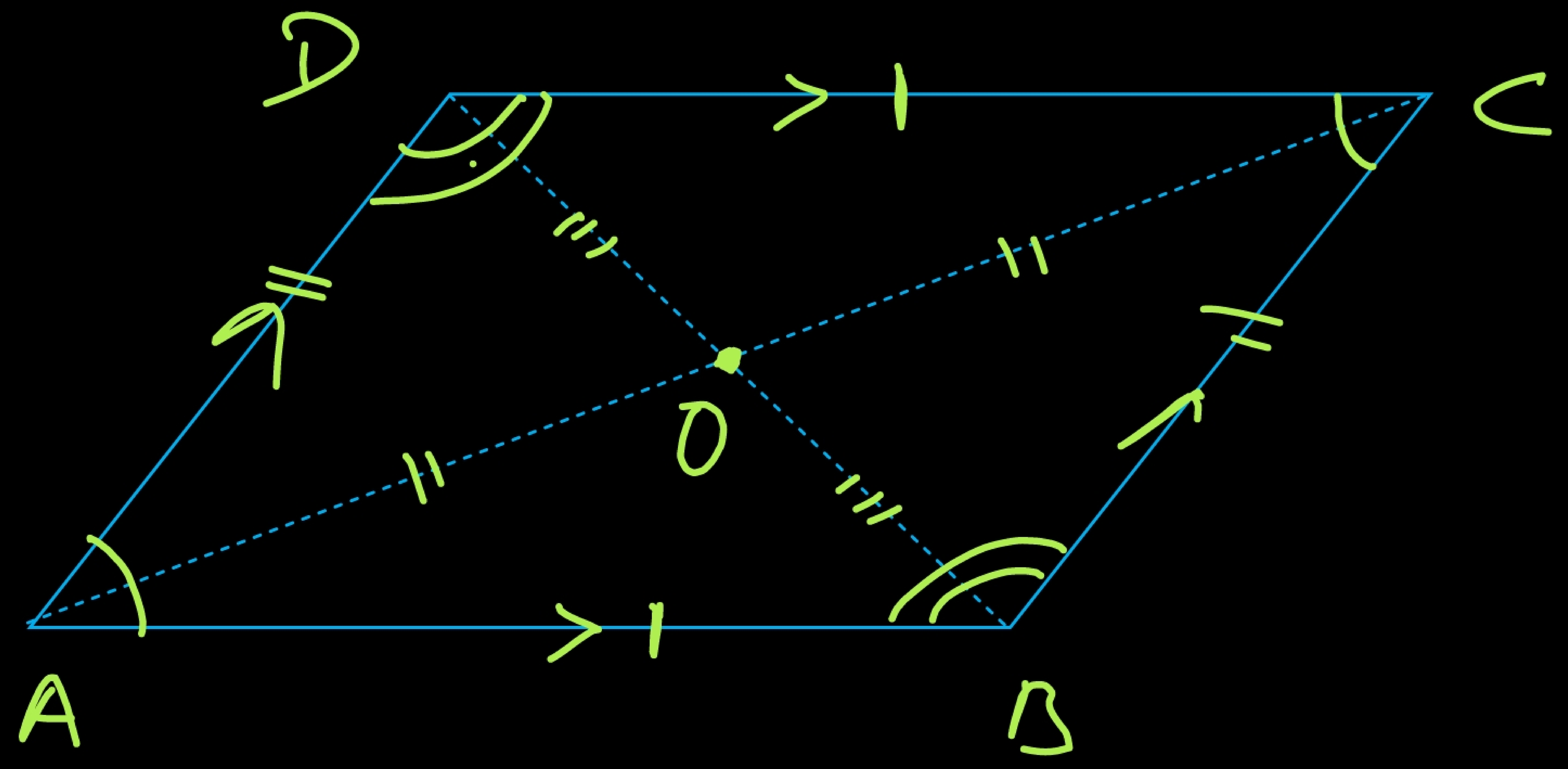
(vii) OPPOSITE ANGLES Two angles of a quadrilateral having no common arm are called its opposite angles.

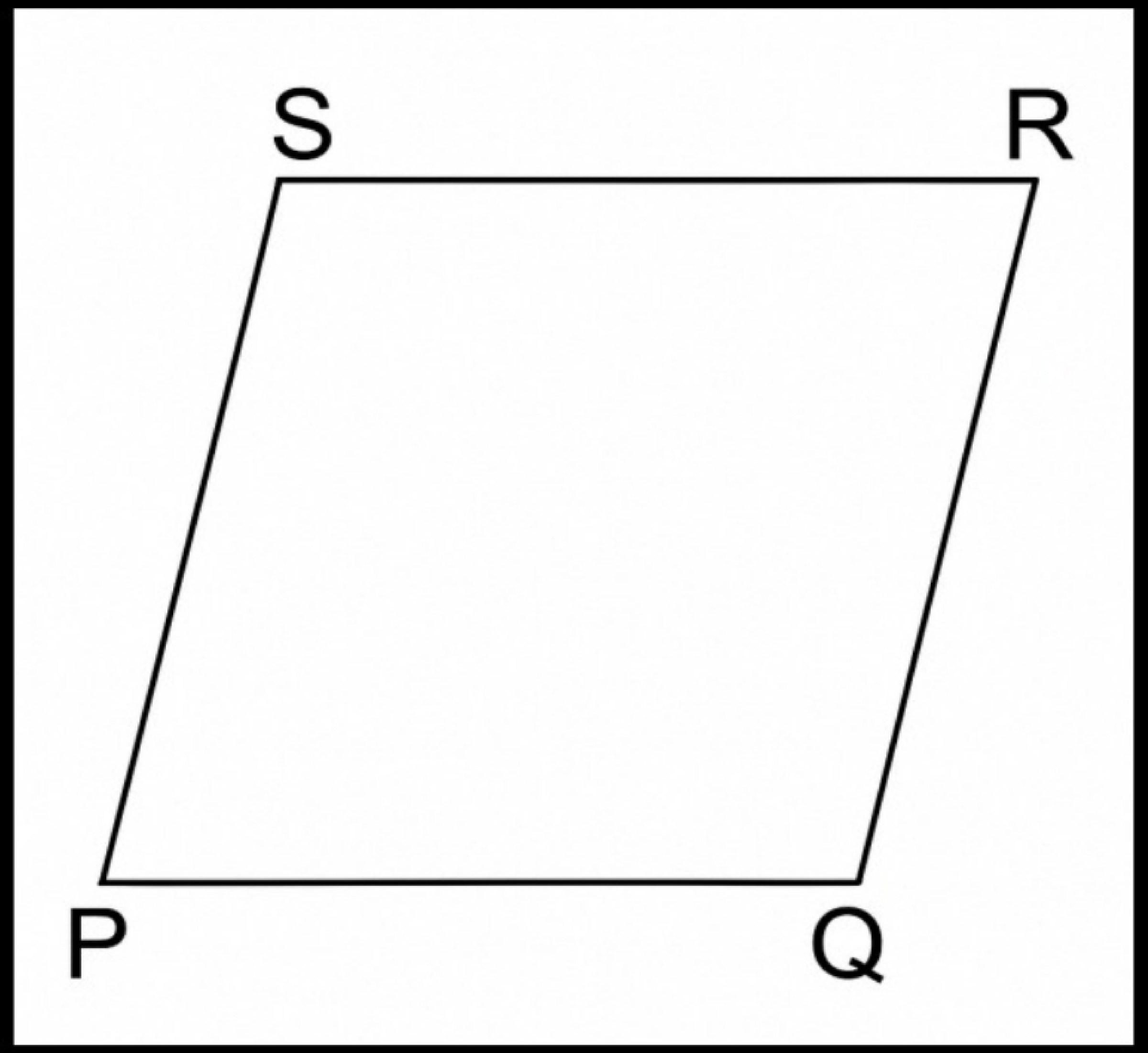
$\angle A, \angle C$ and $\angle B$ $\angle D$ are two pairs of opposite angles of quad. ABCD.

VARIOUS TYPES OF QUADRILATERALS

1. PARALLELOGRAM

A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram, written as ||gm. In |gm PQRS, we have PQ||SR, PS || QR.

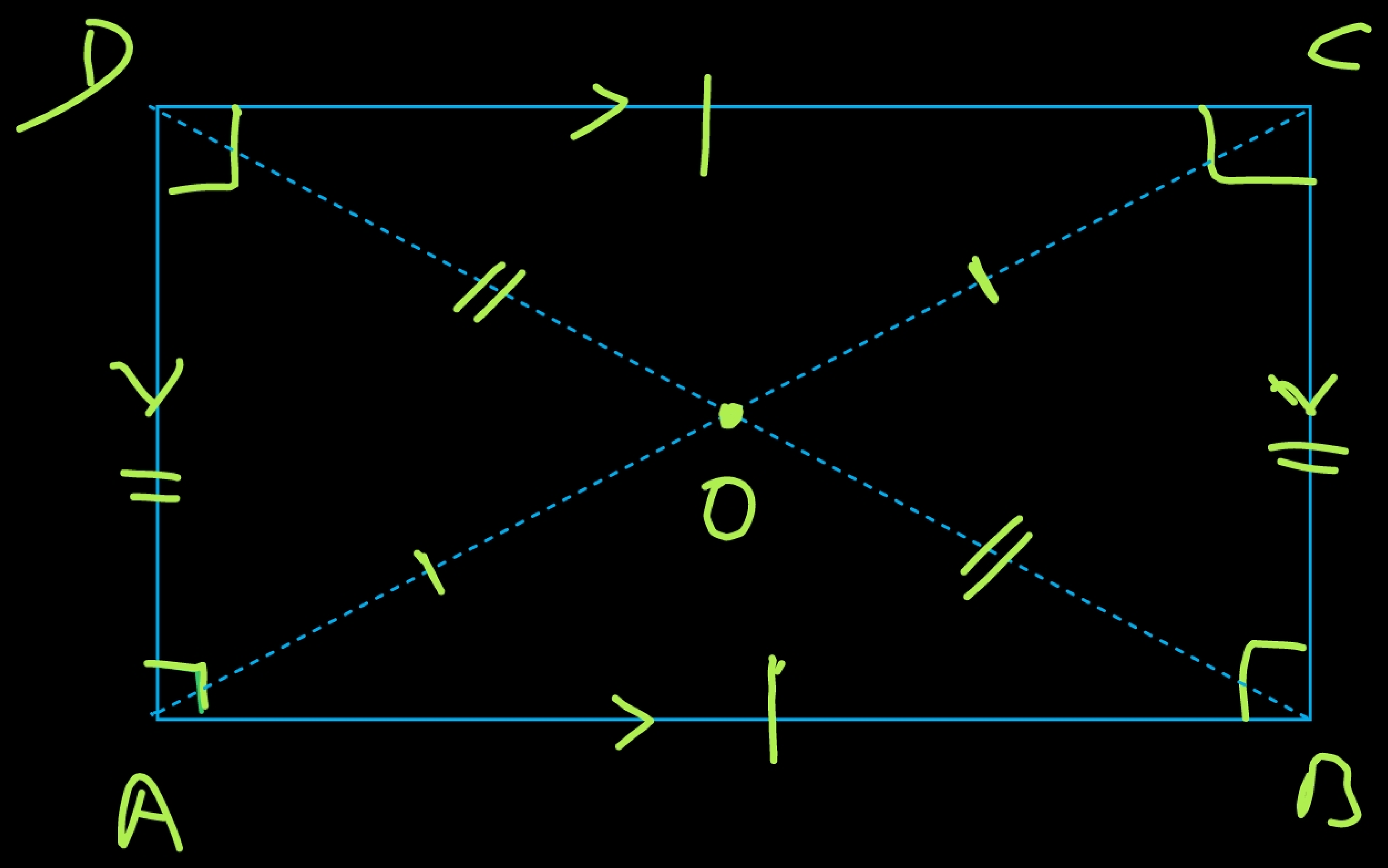


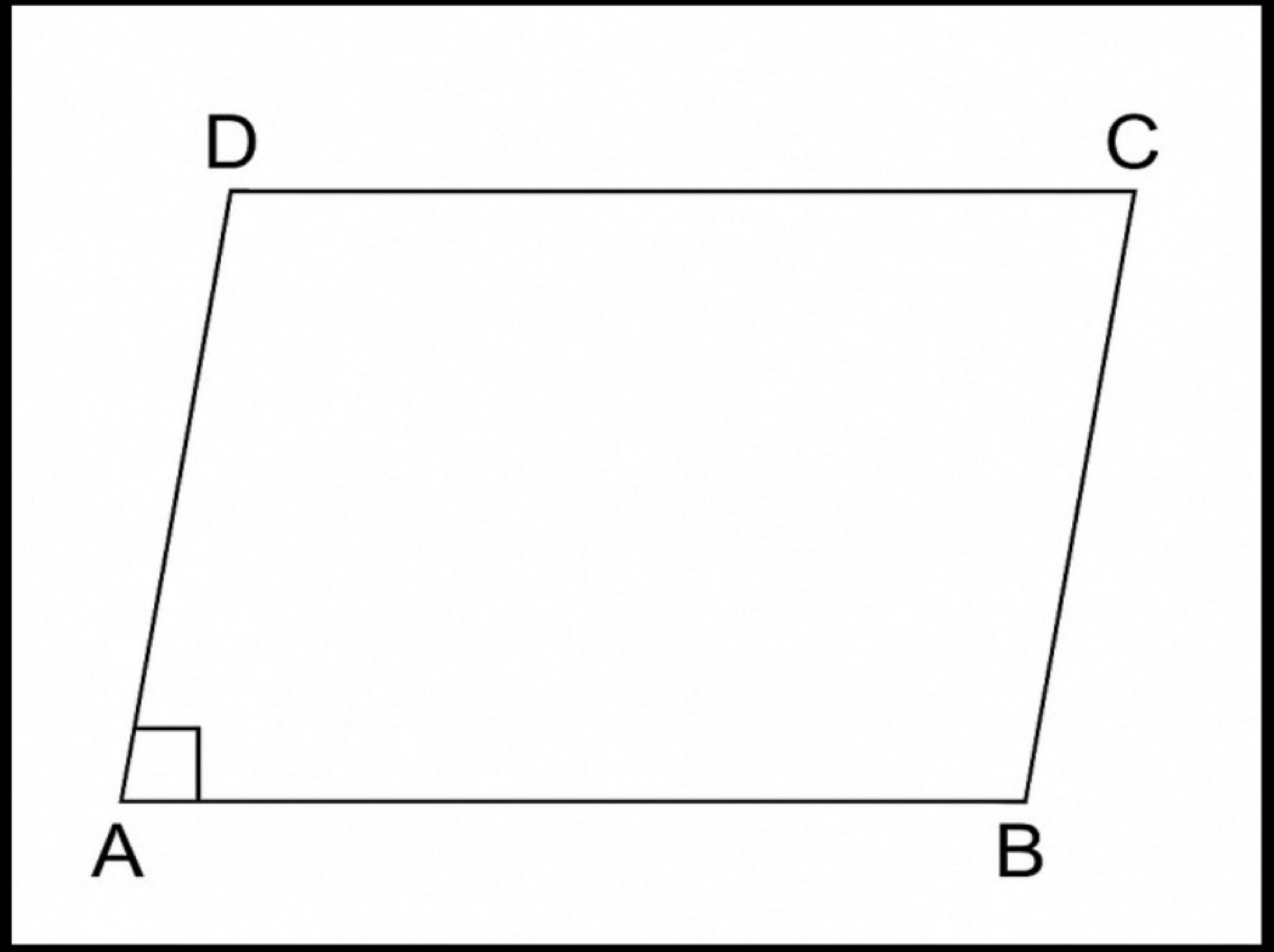


2. RECTANGLE

A parallelogram one of whose angles is 90°, is called a rectangle, written as rect. ABCD, etc.

In rect. ABCD, we have AB| DC, AD || BC and ZA = 90°

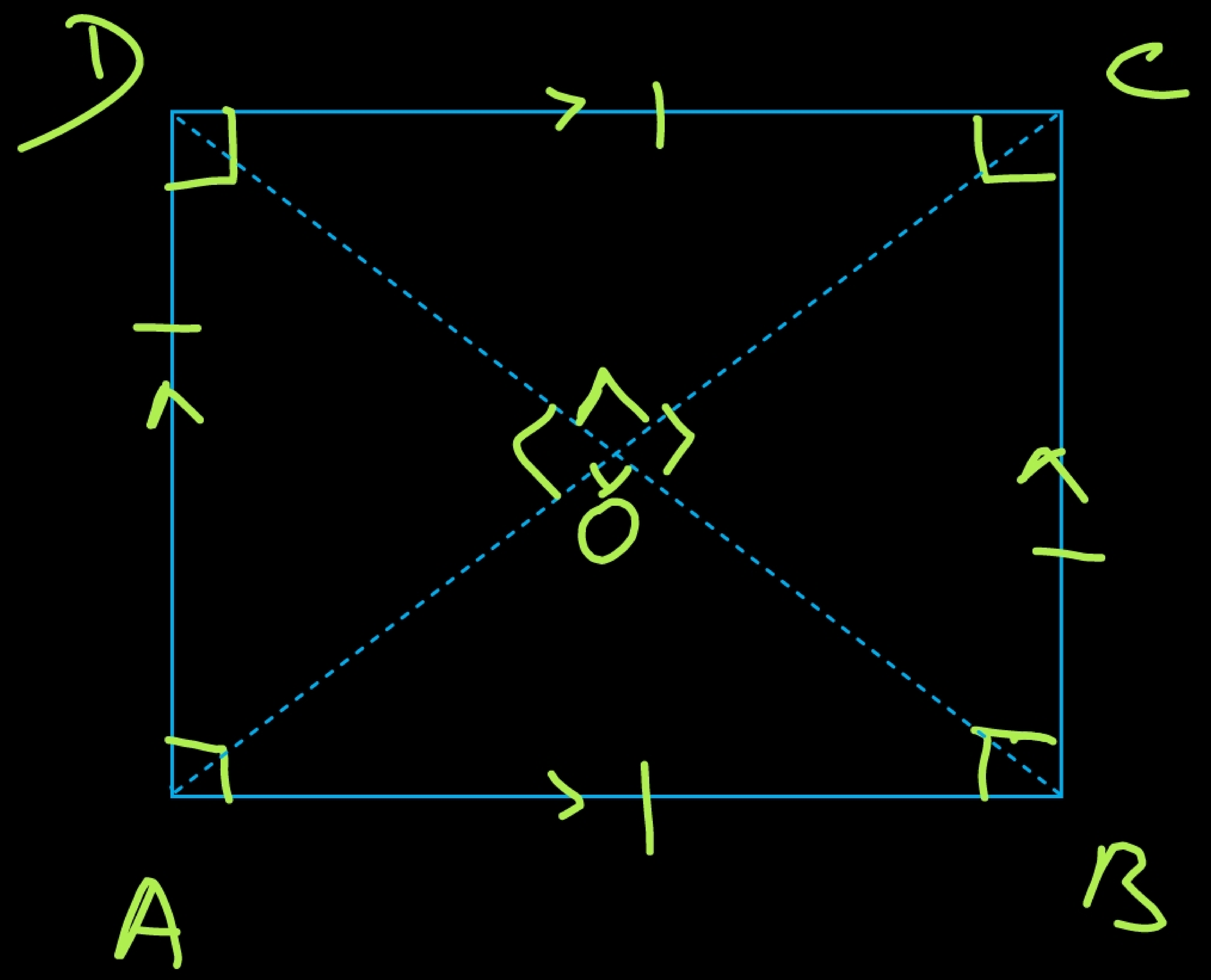


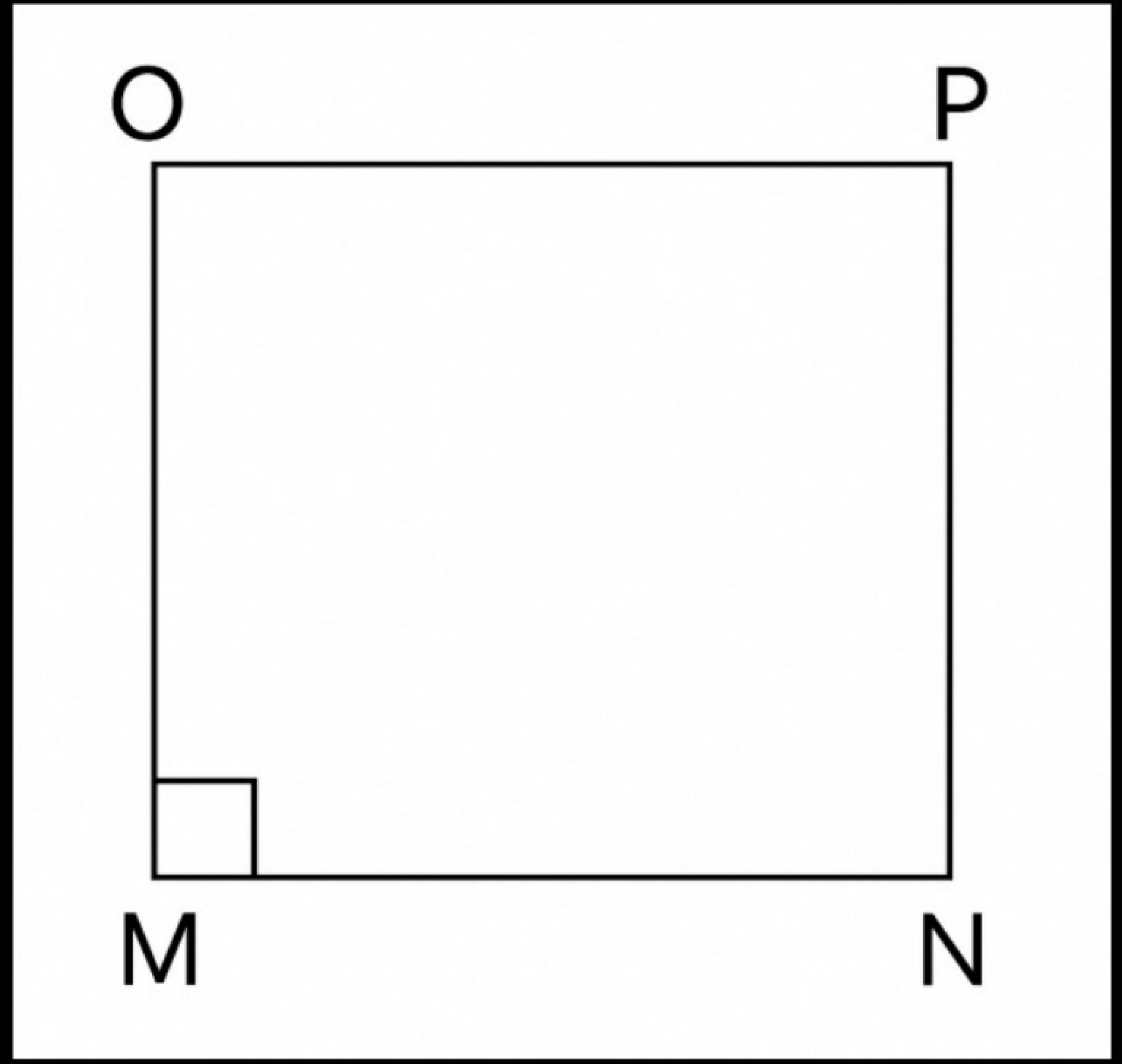


3. SQUARE

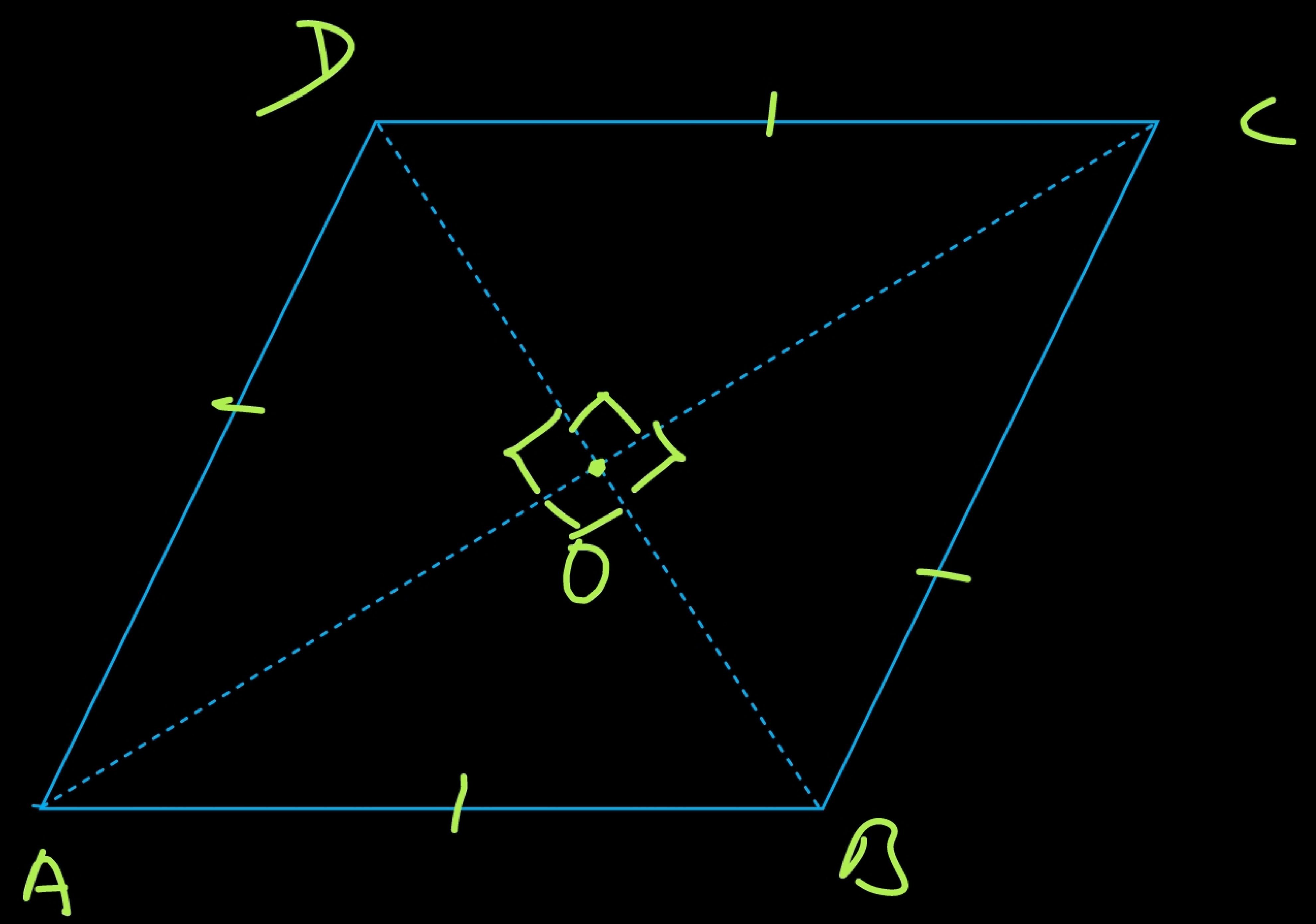
A parallelogram and whrose all sides are equal one of whose angles is 90° is calledasquare. A square is thus a rectangle having all sides equal.

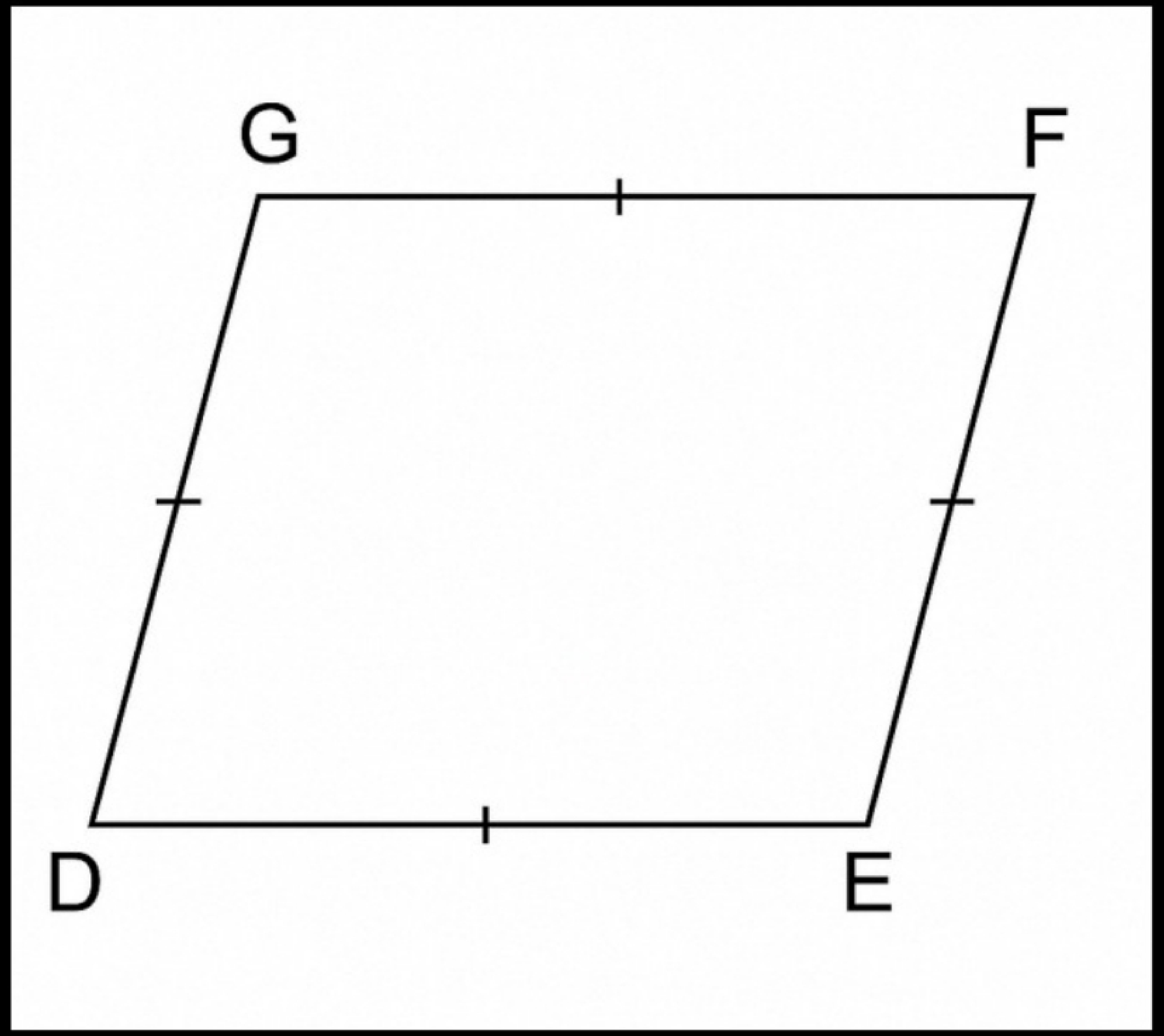
In square MNPQ, we have MN |QP, MQ|| NP and MN = NP = PQ=QM and M = 90°.





4. RHOMBUs A parallelogram having all sides equal is called a rhombus.
In rhombus DEFG, we have of DE |GF, DG|| EF and DE = EF = FG = GD.

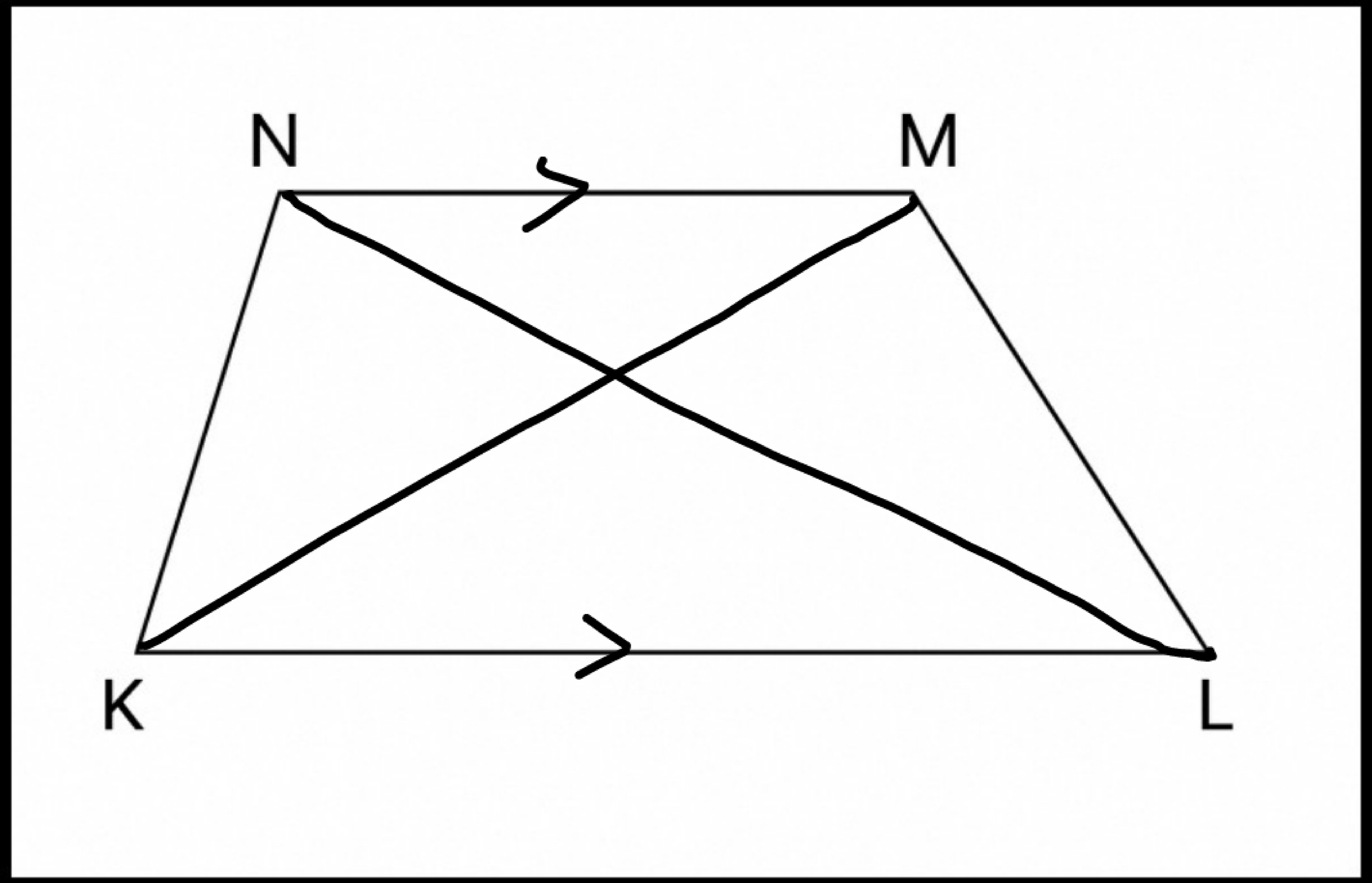




5. TRAPEZIUM

A quadrilateral having one pair of opposite sides parallel is called a trapezium. In trapezium KLMN, we have K a KL| NM.

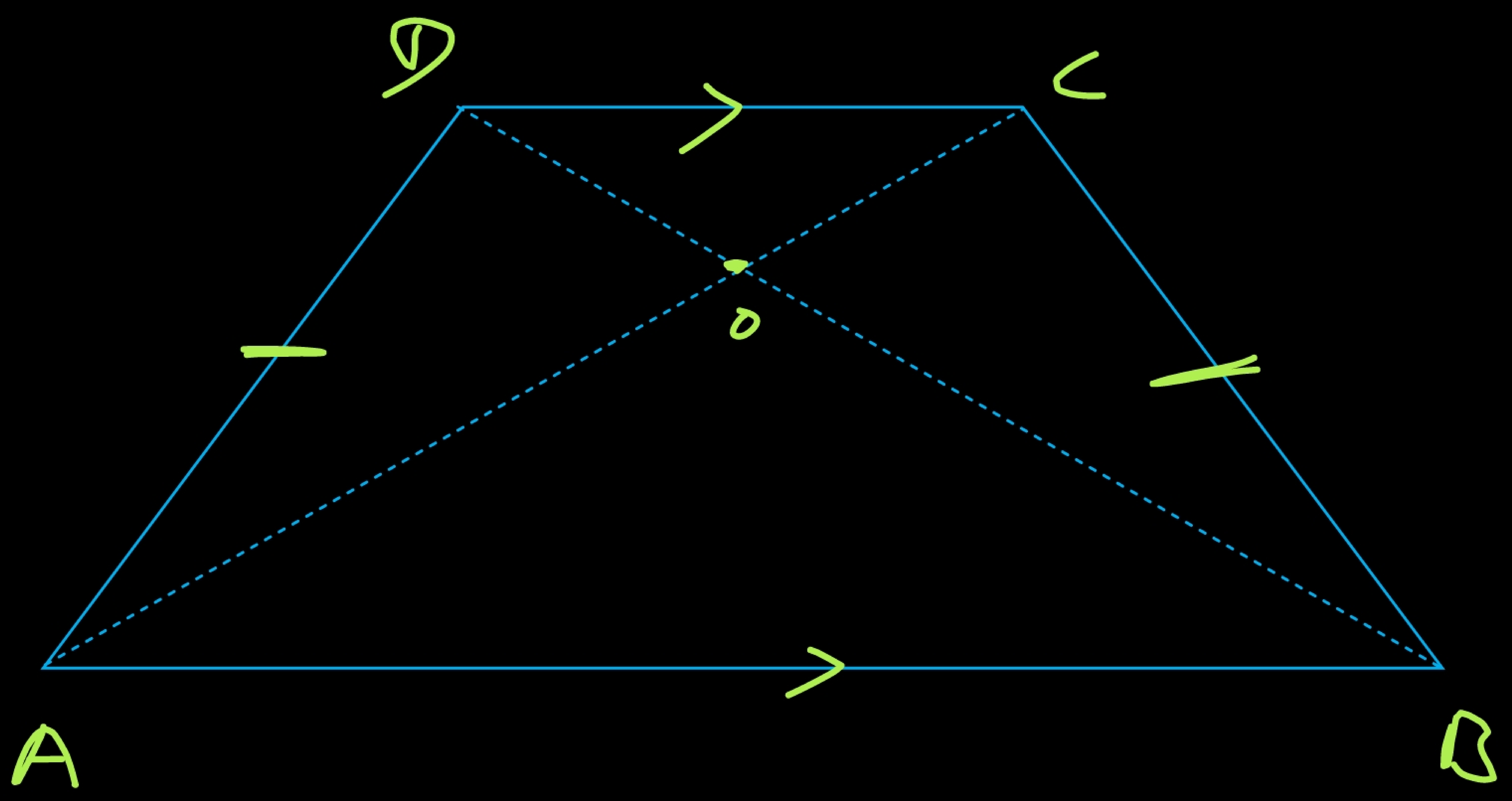
The line segment joining the midpoints of nonparallel sides of a trapezium is called its median.

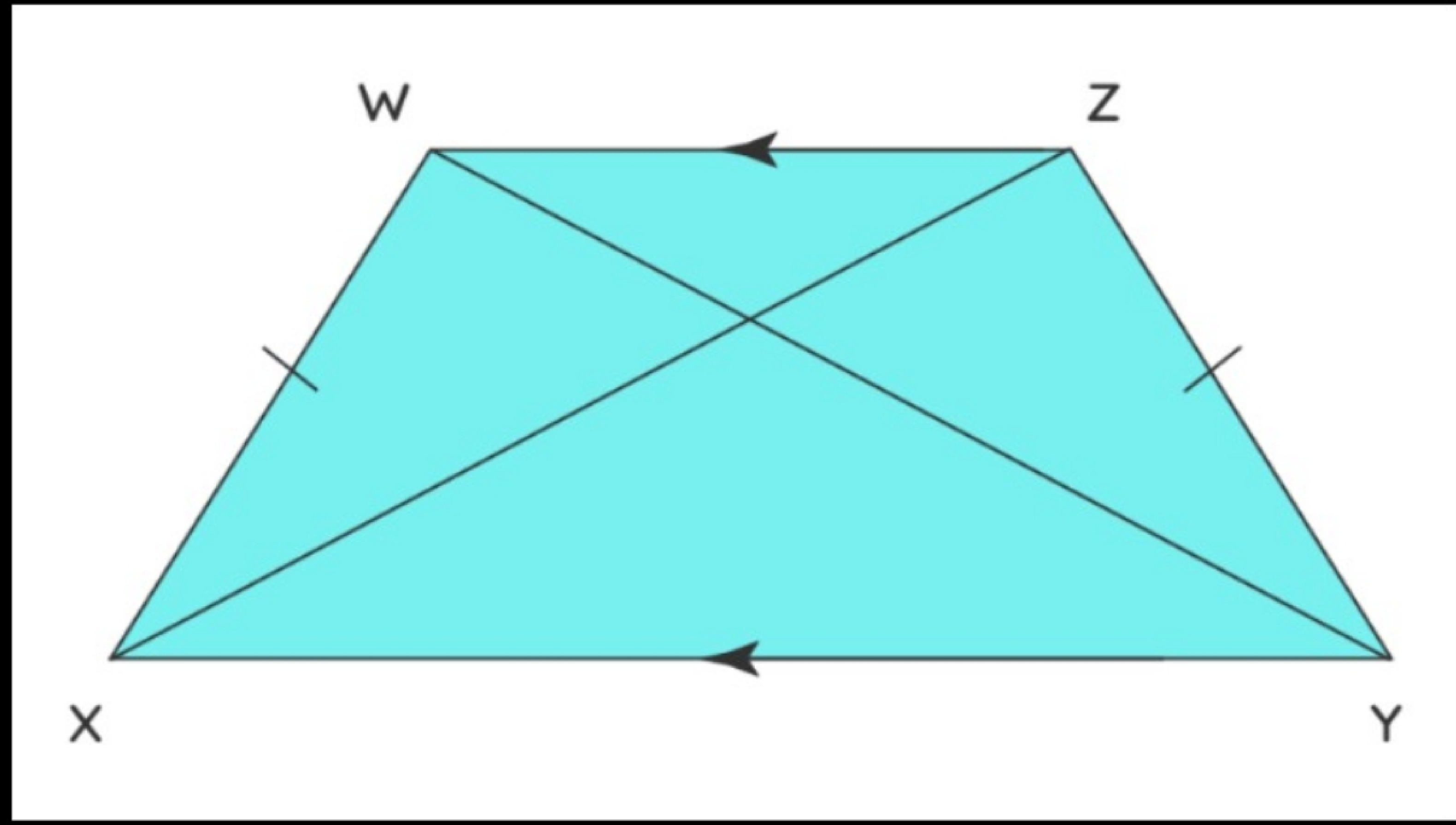


6. ISOSCELES TRAPEZIUM

If the two nonparallel sides of a trapezium are equal then it is called an isosceles trapezium.

In isosceles trapezium PQST, we have PQTS and PT = QS.

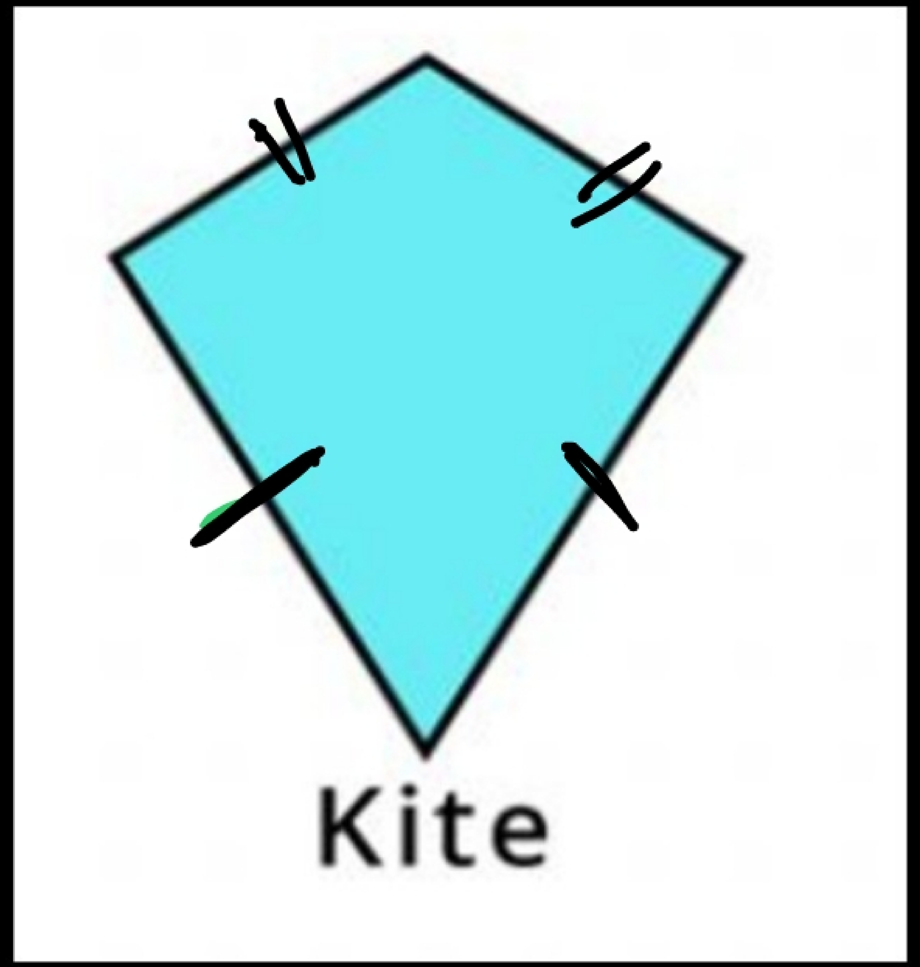




7. KITE

A quadrilateral in which two pairs of adjacent sides are equal is known as a kite.

Quad. ABEF is a kite, in which AB = AF and EB = EF.



From the above definitions it is clear that:

(i) Rectangle, square and rhombus are all parallelograms

(ii) A parallelogram is a trapezium while a trapezium is not parallelogram.

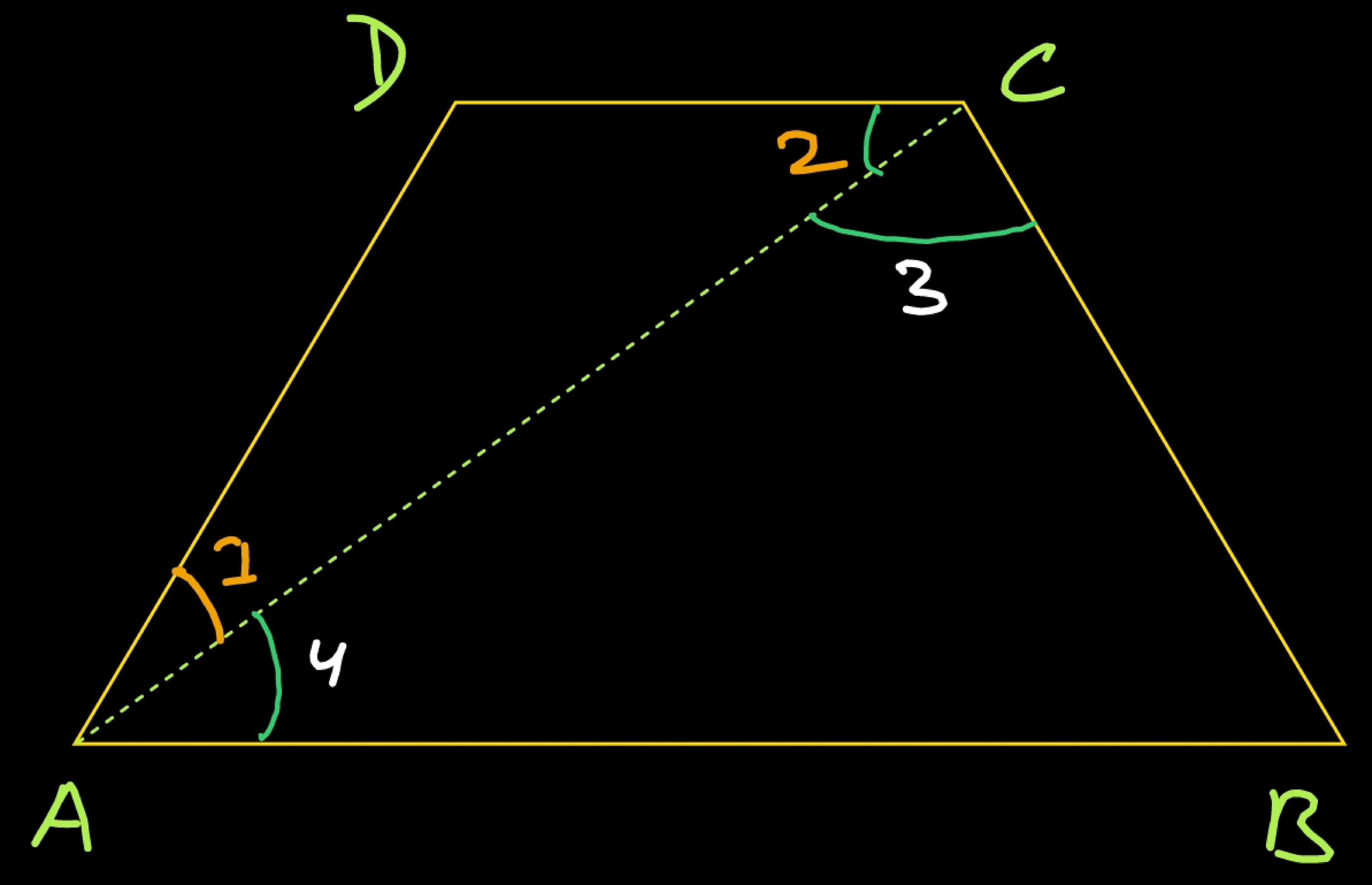
(iii) A square is both a rectangle and a rhombus.

(iv) A kite is not a parallelogram.

(v) A rectangle or a rhombus is not necessarily a square..

THEOREM 1 & Question no. 10

The sum of all the four angles of a quadrilateral is 360°.



Given :- ABCD is a Quadrilateral.

To Prove :- $\angle A + \angle B + \angle C + \angle D = 360°$

Construction :- Join AC

Proof :- In $\triangle ABC$

$\angle 3 + \angle 4 + \angle B = 180°$ {Angle sum property of a $\triangle$} — (1)

In $\triangle ACD$

$\angle 1 + \angle 2 + \angle D = 180°$ — (2) {Angle sum property of a $\triangle$}

On adding eqn (1) & (2)

$\Rightarrow \angle 3 + \angle 4 + \angle B + \angle 1 + \angle 2 + \angle D = 180^\circ + 180^\circ$

$\Rightarrow \angle 1 + \angle 4 + \angle B + \angle 3 + \angle 2 + \angle D = 360^\circ$

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$ Proved


QUADRILATERALS
Rs Aggarwal
Exercise 10A
Class 9th
Example 1 to 8
IOTA Classes
One Shot

EXAMPLE 1

Three angles of a quadrilateral measure 110°, 82° and 68°. Find the measure ofthe fourth angle.

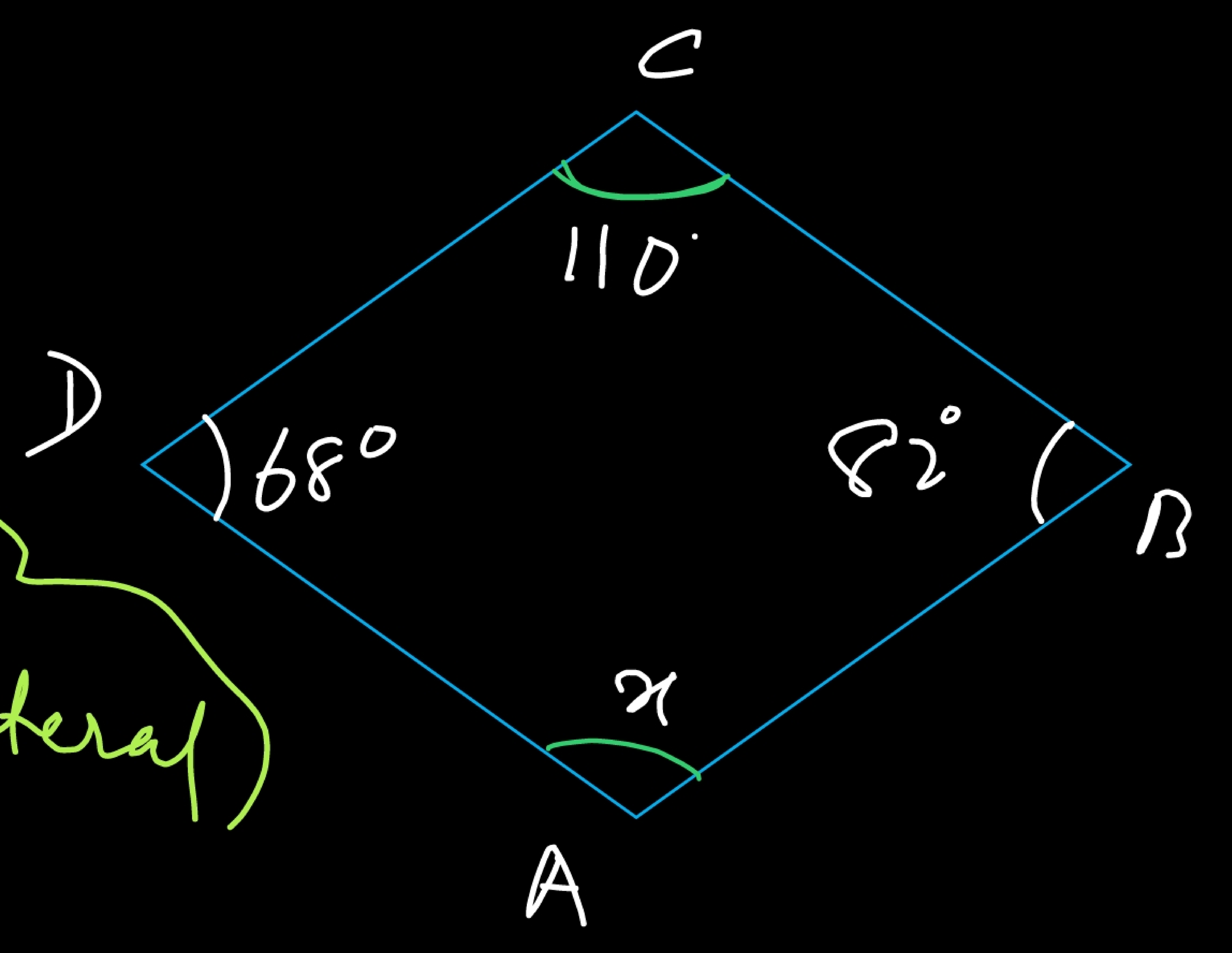
Let the fourth angle be x

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ {Angle sum property of a Quadrilateral}

$\Rightarrow x + 82^\circ + 110 + 68^\circ = 360$

$\Rightarrow x + 260^\circ = 360$

$\Rightarrow \boxed{x = 100^\circ}$



EXAMPLE 2

The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.

Let the ratio be x

$\therefore$ Angles are $= 3x$, $5x$, $9x$ and $13x$

As we know that, Sum of all angle of a Quadrilateral is 360°.

$\Rightarrow 3x + 5x + 9x + 13x = 360^\circ$

$\Rightarrow 30x = 360$

$\Rightarrow x = \dfrac{36\cancel{0}}{\cancel{3}0}$

$\Rightarrow \boxed{x = 12}$

Hence Angles are $= 3x$

$= 3 \times 12$

$= 36^\circ$

2nd angle $= 5x$

$= 5 \times 12$

$= 60^\circ$

3rd Angle $= 9x$

$= 9 \times 12$

$= 108^\circ$

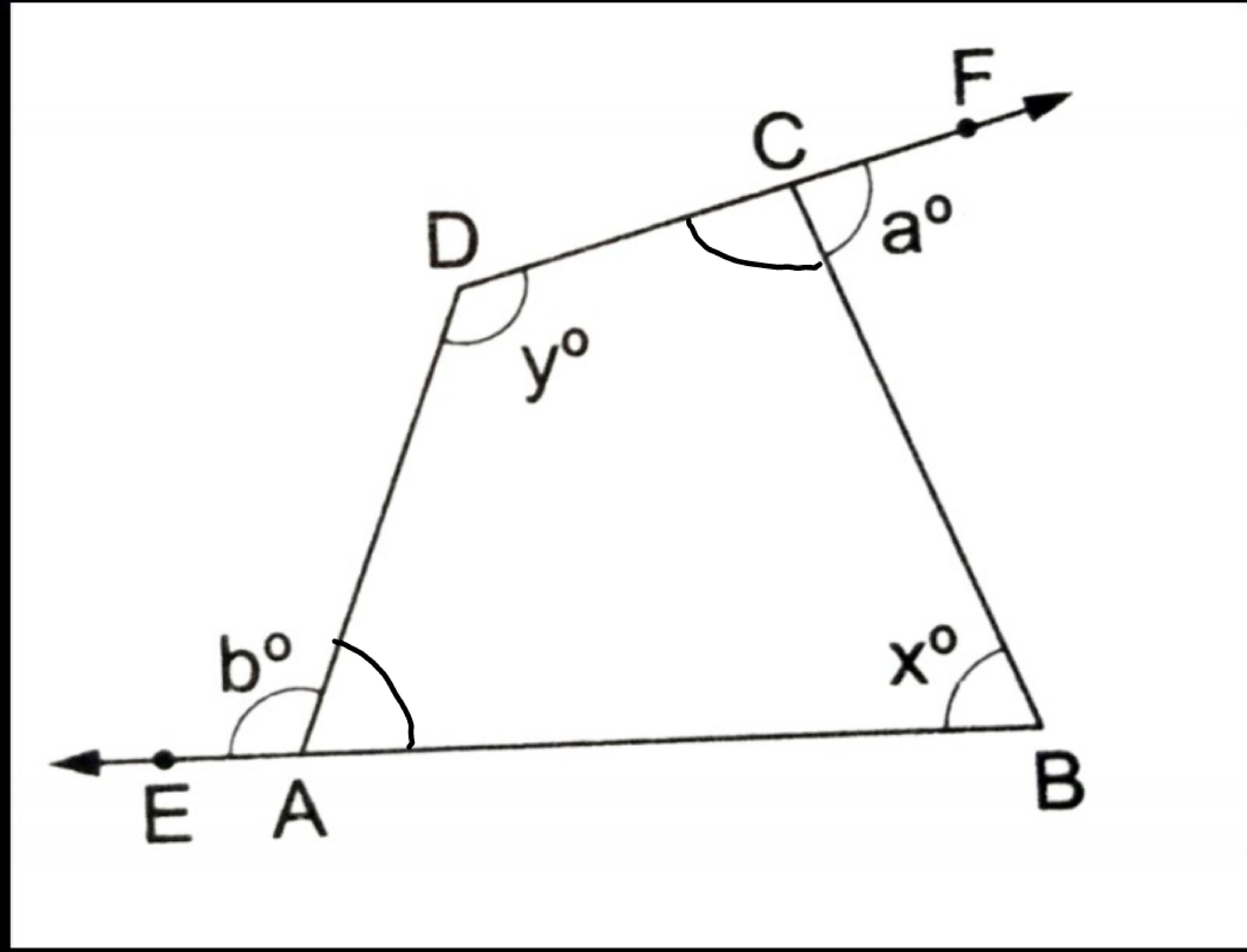
4th angle $= 13x$

$= 13 \times 12$

$= 156^\circ$

EXAMPLE 3

The sides BA and DC of a quadrilateral are produced as shown in the given figure. Prove that x+y= a+b.



Solution:- $b^\circ + \angle DAB = 180^\circ$ {Linear pair}

$\Rightarrow \boxed{\angle DAB = 180^\circ - b^\circ}$

Again

$a^\circ + \angle BCD = 180^\circ$ {Linear pair}

$\boxed{\angle BCD = 180^\circ - a^\circ}$

In Quad ABCD

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ {Angle sum property of a Quad.}

$$\Rightarrow 180^\circ - b^\circ + x^\circ + 180 - a^\circ + y = 360^\circ$$

$$\Rightarrow x + y = \cancel{360} - \cancel{360} + a + b$$

$$\Rightarrow \boxed{x + y = a + b}$$

Proved.

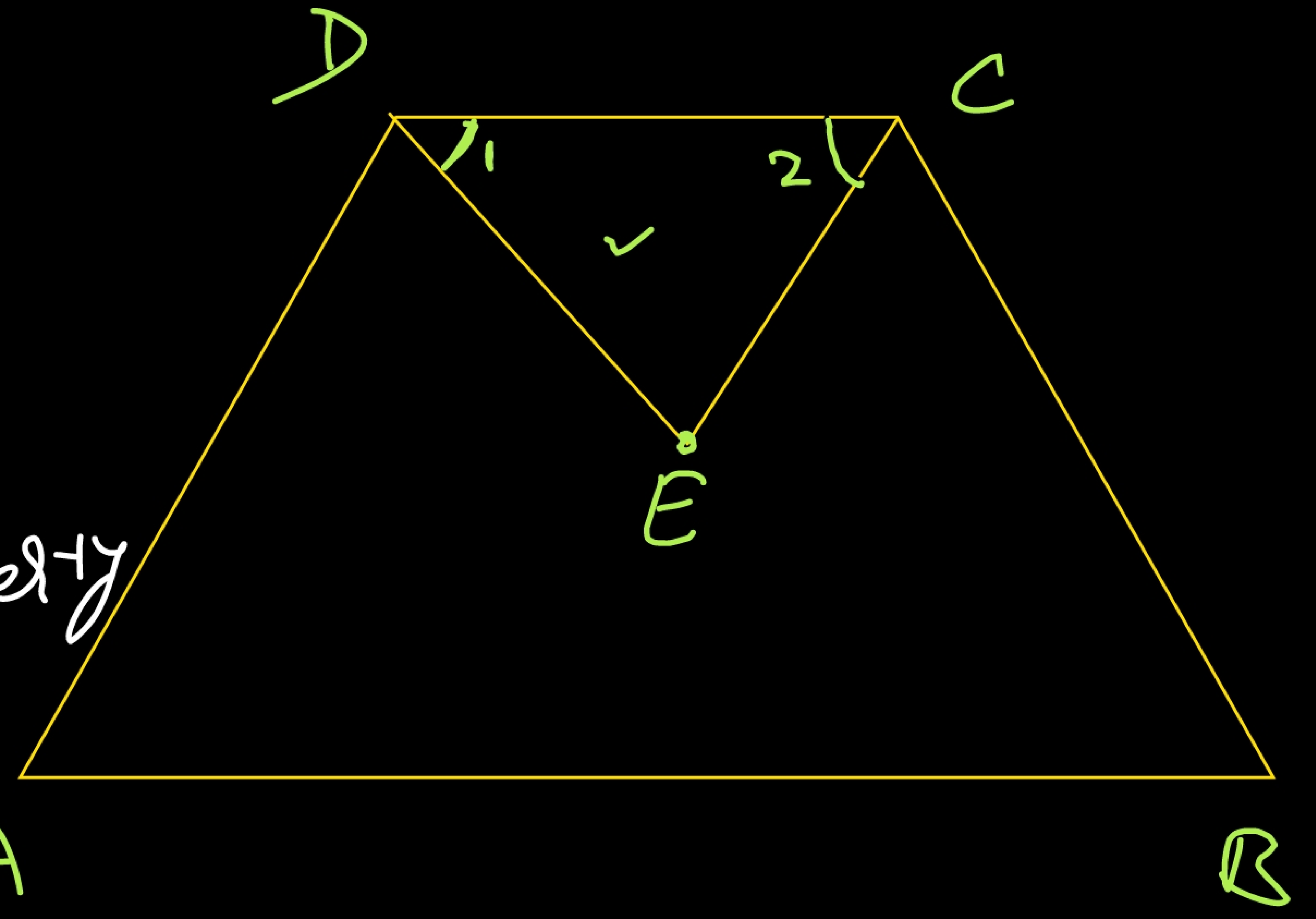
EXAMPLE 4

In a quadrilateral ABCD, the line segments bisecting $\angle C$ and $\angle D$ meet at E. Prove that $\angle A + \angle B = 2\angle CED$.

Solution:- In ΔCED

$\angle 1 + \angle 2 + \angle CED = 180^\circ$

{Angle sum property of a Δ.



$\therefore \frac{1}{2}\angle D + \frac{1}{2}\angle C + \angle CED = 180^\circ$

$\Rightarrow \frac{1}{2}\angle D + \frac{1}{2}\angle C = 180^\circ - \angle CED$ — (i)

In Quad. ABCD

$\angle A + \angle B + \angle C + \angle D = 360°$ { Angle sum property of a Quad. }

Now,

$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C + \frac{1}{2}\angle D = \frac{1}{2} \times 360°$

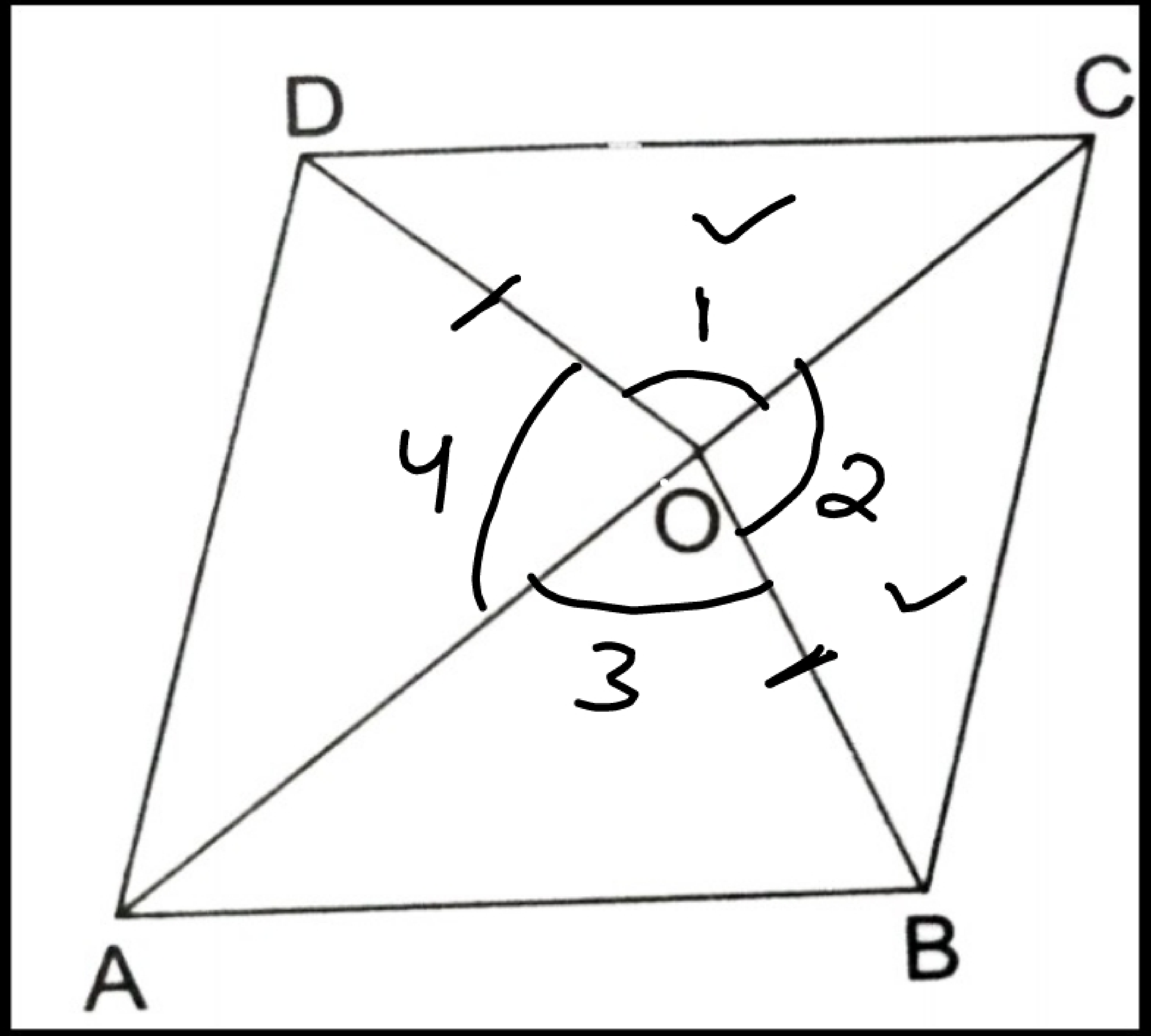
$\Rightarrow \frac{1}{2}(\angle A + \angle B) + \cancel{180°} - \angle CED = \cancel{180°}$

$\Rightarrow \frac{1}{2}(\angle A + \angle B) = \angle CED$

$\Rightarrow \boxed{\angle A + \angle B = 2\angle CED}$ Proved

EXAMPLE 5

In the adjoining figure, a point O is taken inside an equilateral quad. ABCD such that OB = OD. Show that A, O and C are in the same straight line.



In $\triangle COD$ and $\triangle COB$

$OC = OC$ (common)

$CD = BC$ (given)

$OD = OB$ (given)

$\therefore \triangle COD \cong \triangle COB$ {by S.S.S criteria)

$\therefore \boxed{\angle 1 = \angle 2}$

Similarly $\boxed{\angle 3 = \angle 4}$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$

{Sum of all angle around a point is 360°}

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 = 360^\circ$

{$\because \angle 1 = \angle 2$ & $\angle 3 = \angle 4$}

$\Rightarrow 2\angle 1 + 2\angle 4 = 360^\circ$

$\Rightarrow \cancel{2}(\angle 1 + \angle 4) = \cancel{360^\circ}$

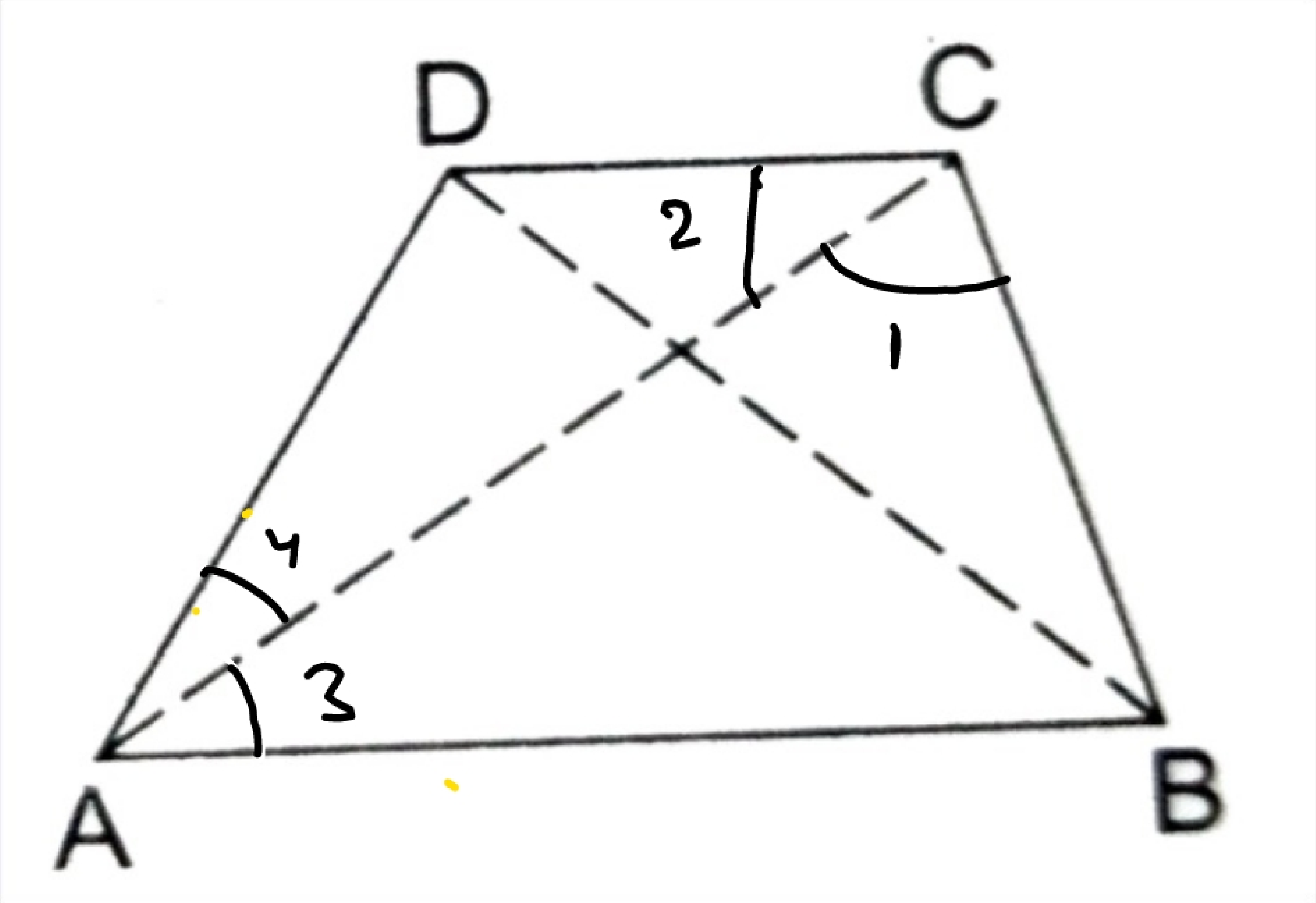
$\Rightarrow \angle 1 + \angle 4 = 180^\circ$

$\therefore$ AOC is a Straight line.

EXAMPLE 6

In the adjoining figure, ABCD is a quadrilateral in which AB is the longest side and CD is the shortest side.

Prove that (i) $\angle C > \angle A$, (ii) $\angle D > \angle B$.



Proof:- In $\triangle ABC$, $AB > BC$ (given)

$\therefore \angle 1 > \angle 3$ {Angle opposite of the longest is greater} —(1)

Now, In $\triangle ACD$, $AD > CD$ {CD is shortest side}

$\therefore \angle 2 > \angle 4$ {Angle opposite of the longest side is greater} —(2)

On adding eqⁿ (1) and (2)

$\Rightarrow \angle 1 + \angle 2 > \angle 3 + \angle 4$

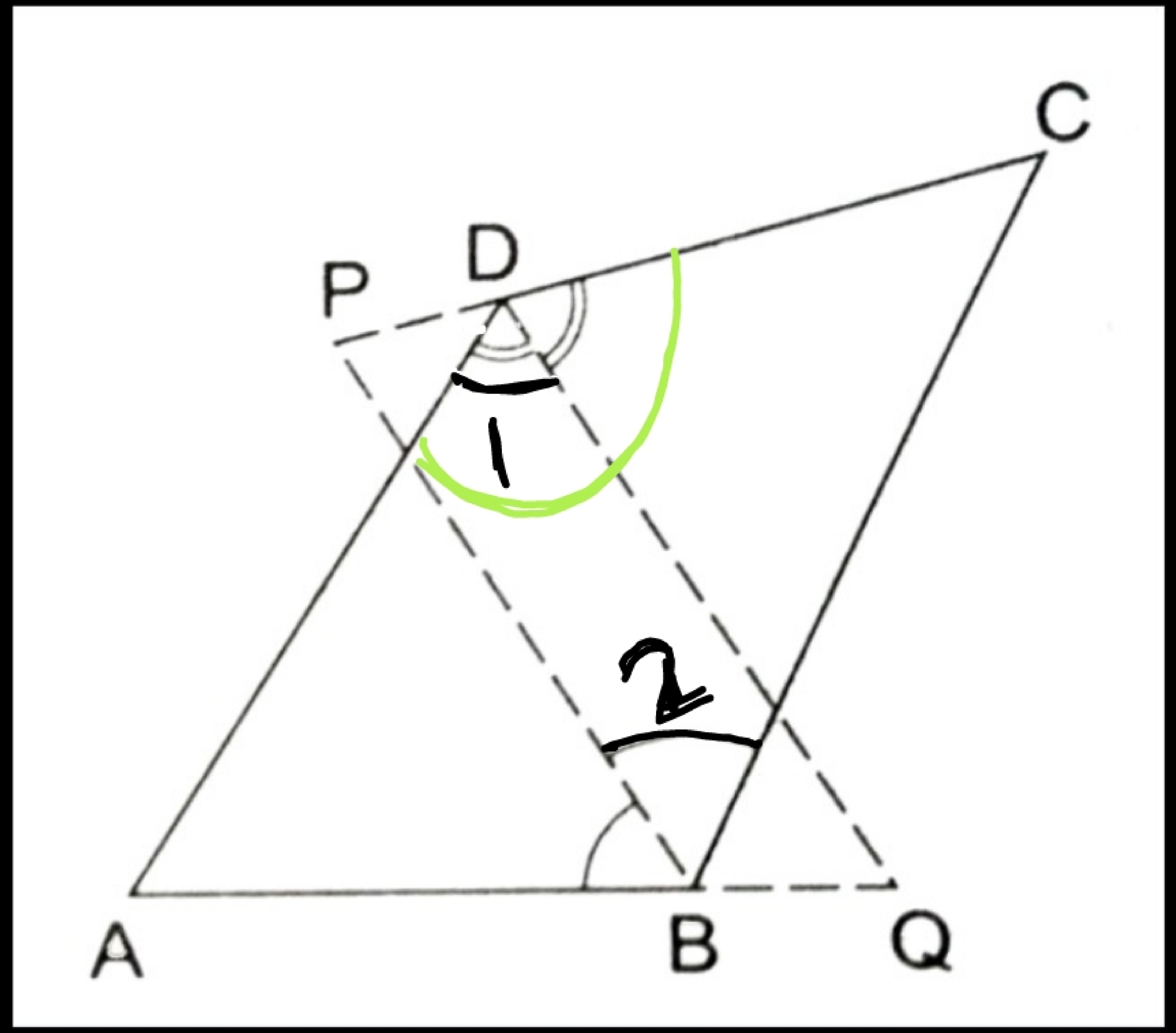
$\Rightarrow \boxed{\angle C > \angle A}$

Similarly $\angle D > \angle B$ Proved.

EXAMPLE 7

In the adjoining figure, the bisectors of $\angle B$ and $\angle D$ of a quadrilateral ABCD meet CD and AB produced at P and Q respectively.

Prove that $\angle P + \angle Q = \frac{1}{2}(\angle B + \angle D)$.



Given:- In adjoining fig BP is the angle bisector of $\angle B$ and DQ is the angle bisector of $\angle D$.

To Prove:- $\angle P + \angle Q = \frac{1}{2}(\angle B + \angle D)$

Proof:- In $\triangle ADQ$

$\angle A + \angle 1 + \angle Q = 180^\circ$ —(1) {Angle Sum property of a $\triangle$}

In $\triangle BCP$

$\angle 2 + \angle C + \angle P = 180^\circ$ — (2) {Angle sum property of $\triangle$}

On adding eqn (1) & (2)

$\angle A + \angle 1 + \angle Q + \angle 2 + \angle C + \angle P = 180^\circ + 180^\circ$

$\Rightarrow \angle A + \frac{1}{2}\angle D + \angle Q + \frac{1}{2}\angle B + \angle C + \angle P = 360^\circ$

On adding $\frac{1}{2}\angle B$ & $\frac{1}{2}\angle D$ both side

$\Rightarrow \angle A + \frac{1}{2}\angle D + \frac{1}{2}\angle D + \angle Q + \frac{1}{2}\angle B + \frac{1}{2}\angle B + \angle C + \angle P = 360^\circ + \frac{1}{2}\angle B + \frac{1}{2}\angle D$

$$\Rightarrow \angle A + \angle B + \angle C + \angle D + \angle P + \angle Q = 360^\circ + \frac{1}{2}(\angle B + \angle D)$$

$$\Rightarrow \cancel{360^\circ} + \angle P + \angle Q = \cancel{360^\circ} + \frac{1}{2}(\angle B + \angle D)$$

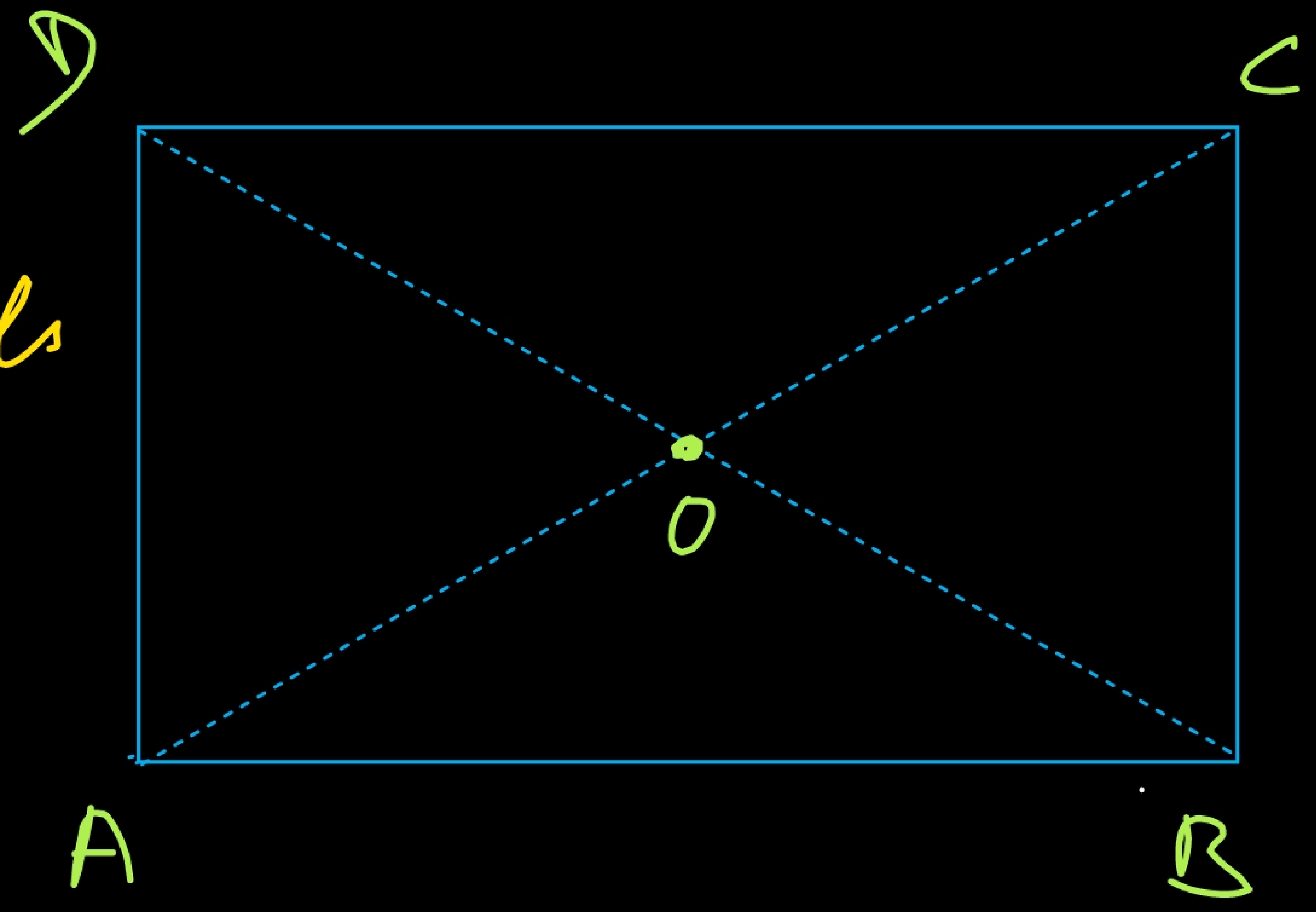
$$\Rightarrow \boxed{\angle P + \angle Q = \frac{1}{2}(\angle B + \angle D)}$$

EXAMPLE 8

If ABCD is a quadrilateral whose diagonals AC and BD intersect at O, prove that

(i) $(AB + BC + CD + DA) > (AC + BD)$,

(ii) $(AB + BC + CD + DA) < 2(AC + BD)$.



Given:- ABCD is a Quad., in which diagonals AC and BD intersect at O.

To Prove:- (1) $AB+BC+CD+DA > AC+BD$

(2) $AB+BC+CD+DA < 2(AC+BD)$

Proof:- In ΔABC

$AB+BC > AC$ — (1) {The sum of two sides of a Δ is greater than 3rd side}

Now, In $\triangle ACD$

$CD + DA > AC$ — (2) {Same reason)

In $\triangle ABD$,

$DA + AB > BD$ — (3) {Same reason}

In $\triangle BCD$

$BC + CD > BD$ — (4) {Same reason)

On adding eq^n (1), (2), (3) and (4)

$$\Rightarrow AB + BC + CD + DA + DA + AB + BC + CD > AC + AC + BD + BD$$

$$\Rightarrow 2AB + 2BC + 2CD + 2DA > 2AC + 2BD$$

$$\Rightarrow \cancel{2}(AB + BC + CD + DA) > \cancel{2}(AC + BD)$$

$$\Rightarrow \boxed{AB + BC + CD + DA > AC + BD}$$

Now,

(2) In ΔAOB

$\Rightarrow AO + BO > AB$ — (1) { The sum of two side of a Δ is greater than the 3rd side }

In ΔBOC

$BO + OC > BC$ — (2) { The sum of two side of a Δ is greater than the 3rd side }

In ΔCOD

$OC + OD > CD$ — (3) { The sum of two side of a Δ is greater than the 3rd side }

In ΔDOA

$AO + OD > AD$ — (4) (")

On adding eqn 1, 2, 3 and 4

$$AO + BO + BO + CO + CO + OD + AO + OD > AB + BC + CD + DA$$

$$\Rightarrow 2AO + 2BO + 2CO + 2OD > AB + BC + CD + DA$$

$$\Rightarrow 2AO + 2CO + 2BO + 2OD > AB + BC + CD + DA$$

$$\Rightarrow 2(AO + CO + BO + OD) > AB + BC + CD + DA$$

$$\Rightarrow 2(AC + BD) > AB + BC + CD + DA$$

$$\Rightarrow \boxed{AB + BC + CD + DA < 2(AC + BD)}$$


QUADRILATERALS
Rs Aggarwal
Class 9th
Exercise 10A
Questions
1 to 10
One Shot
IOTA Classes

EXERCISE 10 A

1. Three angles of a quadrilateral are 75°, 90° and 75°. Find the measure of the fourth angle.

Let the fourth angle be x

As, we know that the Sum of all angle of a Quadrilateral is 360

$\therefore 75^\circ + 90^\circ + 75^\circ + x = 360^\circ$

$\Rightarrow x = 360 - 240$

$\Rightarrow \boxed{x = 120^\circ}$

iOTA Classes सत्यमेव जयते

2. The angles of a quadrilateral are in the ratio 2 : 4 : 5 : 7. Find the angles.

Let the ratio be x

then the angles are $= 2x, 4x, 5x$ and $7x$

As we know that the sum of all angle of a Quad. is 360°

$\Rightarrow 2x + 4x + 5x + 7x = 360^\circ$

$\Rightarrow 18x = 360$

$\Rightarrow \boxed{x = 20}$

$\therefore$ Angles are $= 2x$

$= 2 \times 20$

$= 40$

2nd Angle $= 4x$

$= 4 \times 20$

$= 80$

3rd Angle $= 5x$

$= 5 \times 20$

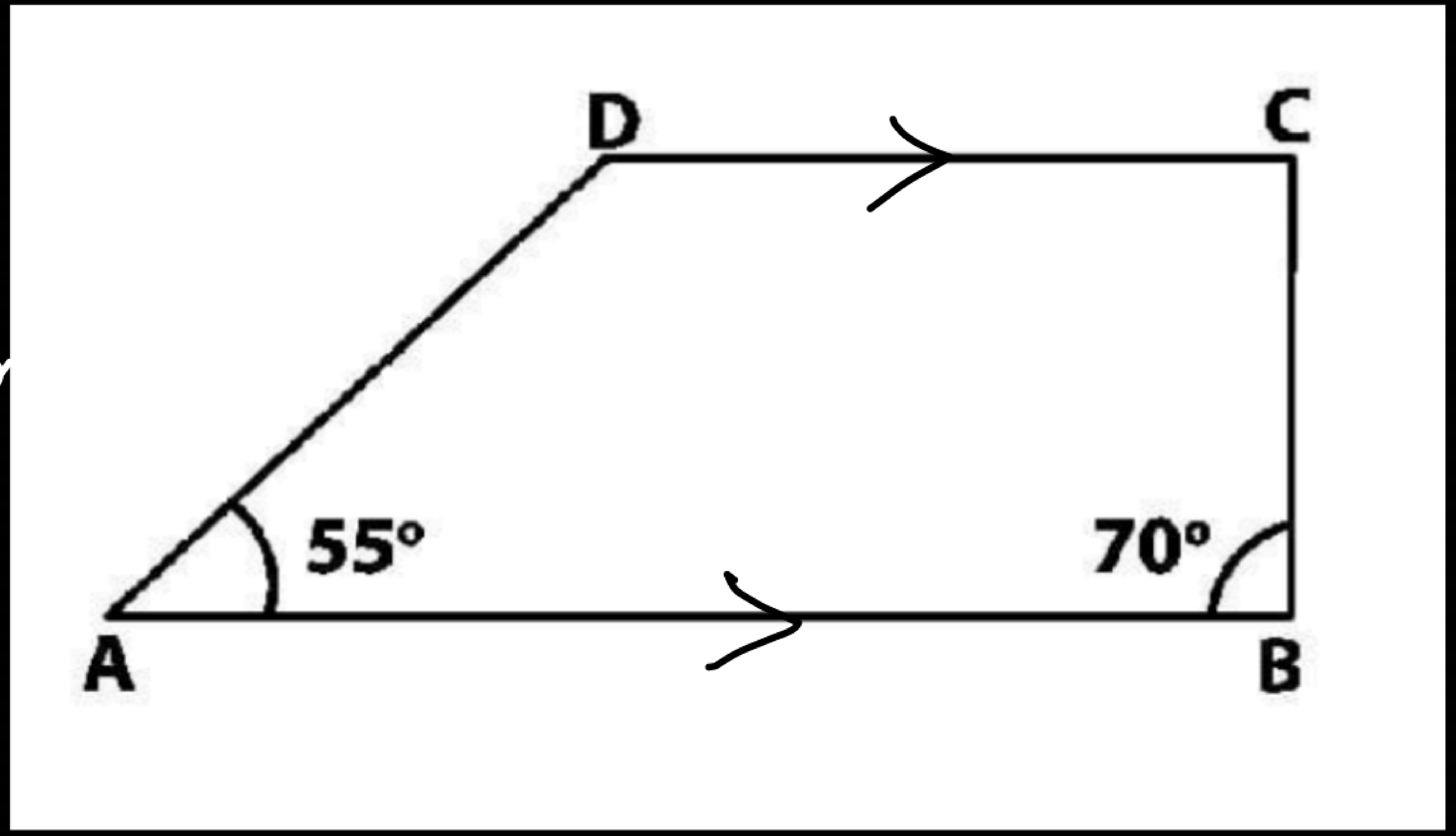
$= 100$

fourth angle $= 7x$

$= 7 \times 20$

$= 140$

3. In the adjoining figure, ABCD is a trapezium in which AB || DC. If $\angle A = 55^\circ$ and $\angle B = 70^\circ$, find $\angle C$ and $\angle D$.



Solution:- AB || CD and BC is a transversal

$\therefore \angle B + \angle C = 180^\circ$ {sum of Co. interior angle

$\Rightarrow 70 + \angle C = 180$

$\Rightarrow \angle C = 110^\circ$

Now,

AB || CD and AD is a transversal

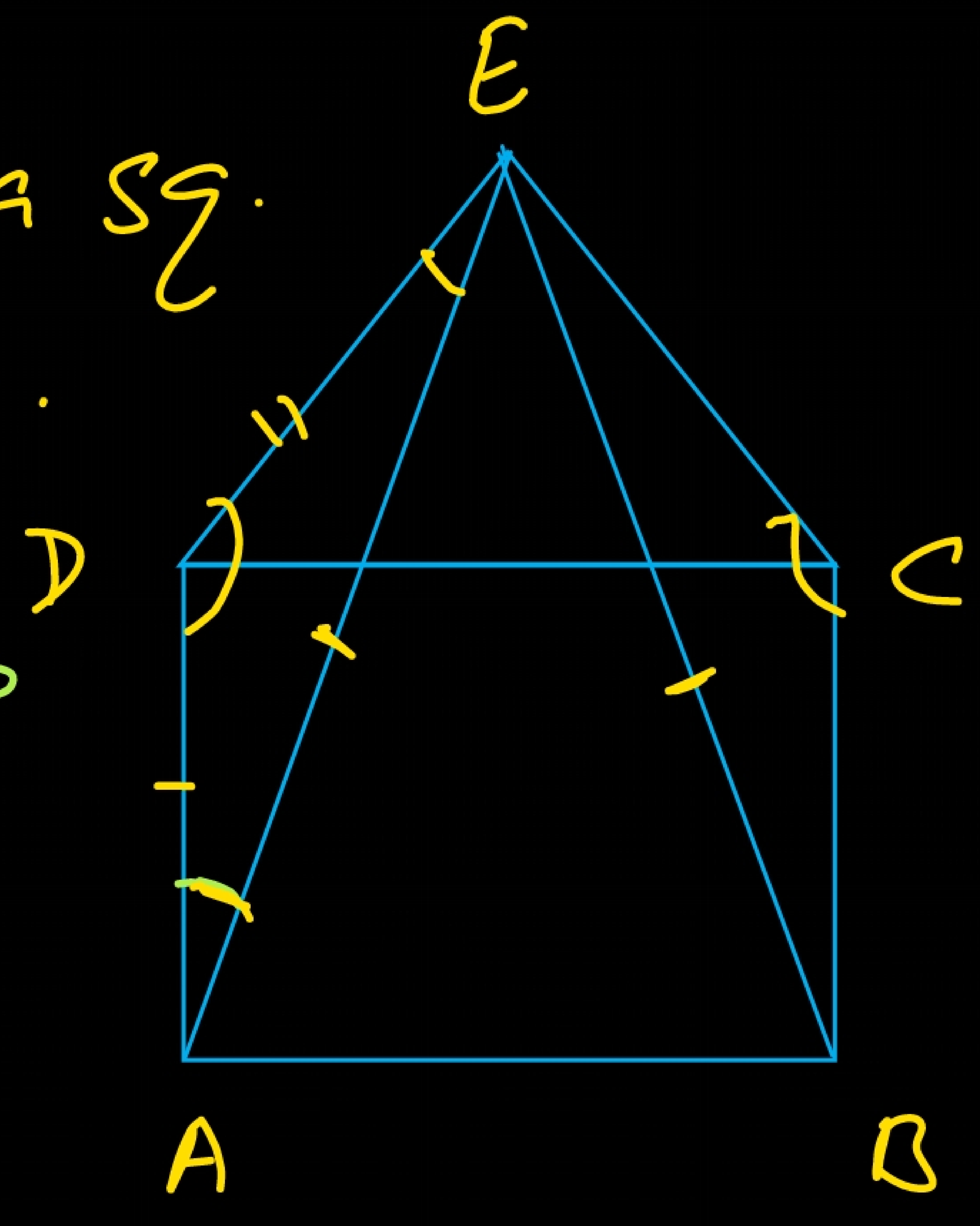
$\angle A + \angle D = 180^\circ$ {sum of Co. interior angles}

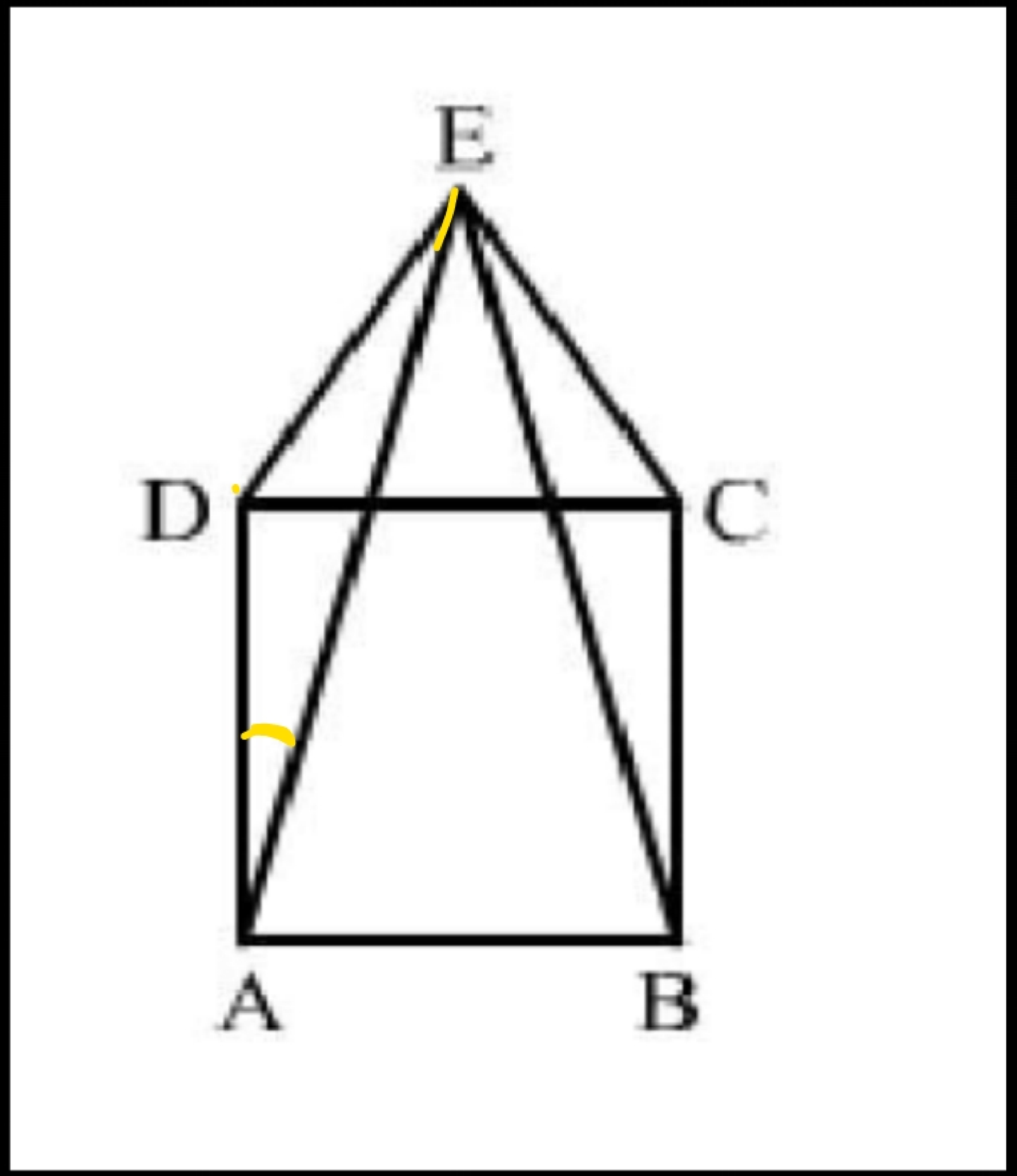
$\Rightarrow 55 + \angle D = 180^\circ$

$\Rightarrow \angle D = 125^\circ$

4. In the adjoining figure, ABCD is a square and $\triangle$EDC is an equilateral triangle. Prove that (i) AE = BE, (ii) $\angle$DAE = 15°.

Given:- In the adjoining fig ABCD is a sq. and EDC is an equilateral Δ.





To Prove:- (i) AE = BE (ii) $\angle$DAE = 15°

Proof:- In ΔADE & ΔBCE

AD = BC (Sides of a square)

DE = EC (Sides of an equilateral Δ)

$\angle$ADE = $\angle$BCE {each equal to 150°}

$\therefore \triangle ADE \cong \triangle BCE$ (by S.A.S criteria)

Hence $AE = BE$

Now $AD = CD$ — ① (Side of square)

$CD = DE$ — ② (Sides of an equilateral $\triangle$)

from eqn ① & ②

$$AD = DE$$

$\therefore \angle DAE = \angle DEA$ {Angles ~~o~~pposite to equal sides of a $\triangle$ are equal.)

In $\triangle DAE$

$\angle ADE + \angle DAE + \angle DEA = 180$ {angle sum property of a $\triangle$}

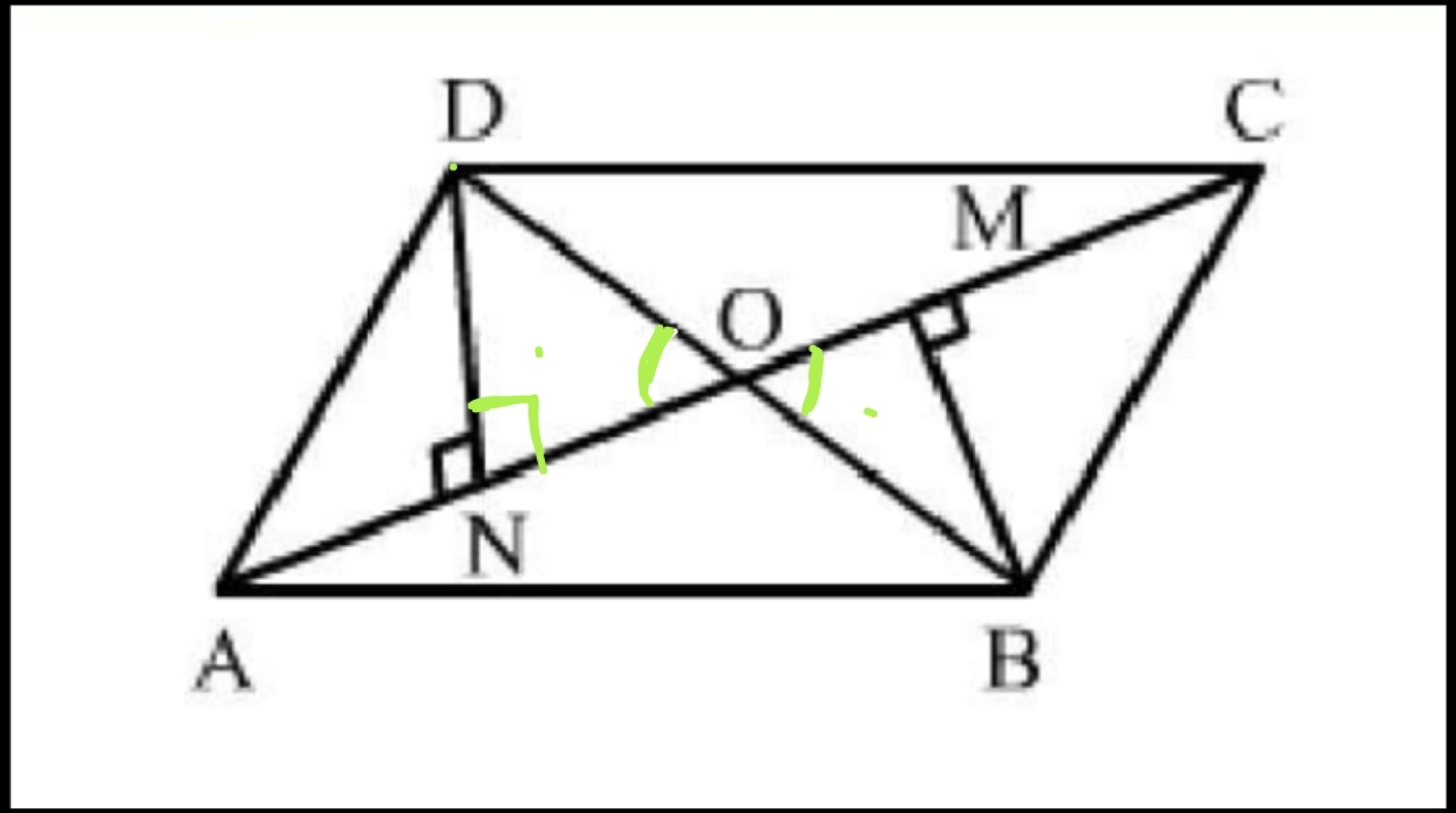
$\Rightarrow 150 + \angle DAE + \angle DAE = 180^\circ$ $\{\because \angle DAE = \angle DEA\}$

$\Rightarrow 2\angle DAE = 180^\circ - 150$

$\Rightarrow 2\angle DAE = 30$

$\Rightarrow \boxed{\angle DAE = 15^\circ}$ Proved,,

5. In the adjoining figure, $BM \perp AC$ and $DN \perp AC$. If $BM = DN$, prove that AC bisects BD.



Given :- In the adjoining fig $BM \perp AC$ and $DN \perp AC$ and $BD = DN$

To Prove :- $OB = OD$

Proof :- In ΔDON and ΔBOM

$DN = BM$ (given)

$\angle DON = \angle BOM$ (vertically opposite Angle)

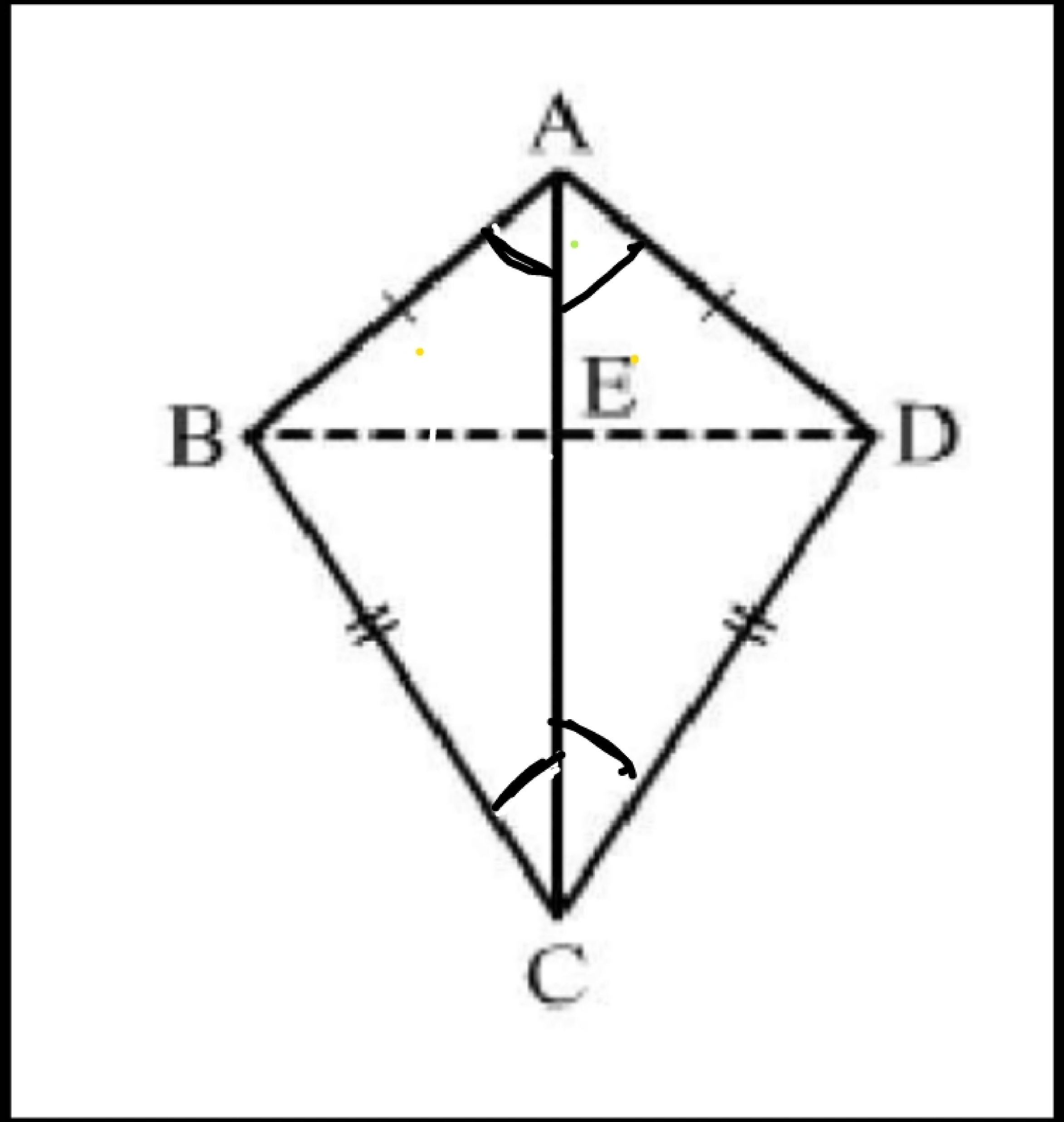
$\angle DNO = \angle BMO$ {each equal to $90°$}

$\therefore \Delta DON \cong \Delta BOM$ (by A.A.S)

$\therefore \quad OB = OD$

Hence AC is the bisector of BD.

6. In the given figure, ABCD is a quadrilateral in which AB = AD and BC = DC. Prove that (i) AC bisects ∠A and ∠C, (ii) BE = DE, (iii) ∠ABC = ∠ADC.



Given:- ABCD is a quadrilateral $AB = AD$ and $BC = DC$

To Prove:- (i) AC bisects $\angle A$ and $\angle C$ (ii) $BE = DE$

(iii) $\angle ABC = \angle ADC$

Proof:- In ΔABC and ΔADC

$AB = AD$ (given)

$BC = DC$ (given)

$AC = AC$ (common)

$\therefore \Delta ABC \cong \Delta ADC$ (by S.S.S criteria)

$\therefore \angle BAC = \angle DAC$

$\angle BCA = \angle DCA$ } by C.P.C.T

$\therefore$ AC bisects $\angle A$ and $\angle C$.

(3) $\therefore \angle ABC = \angle ADC$ (by C.P.C.T)

In $\triangle AEB$ and $\triangle ADE$

$AB = AD$ (given)

$AE = AE$ (common)

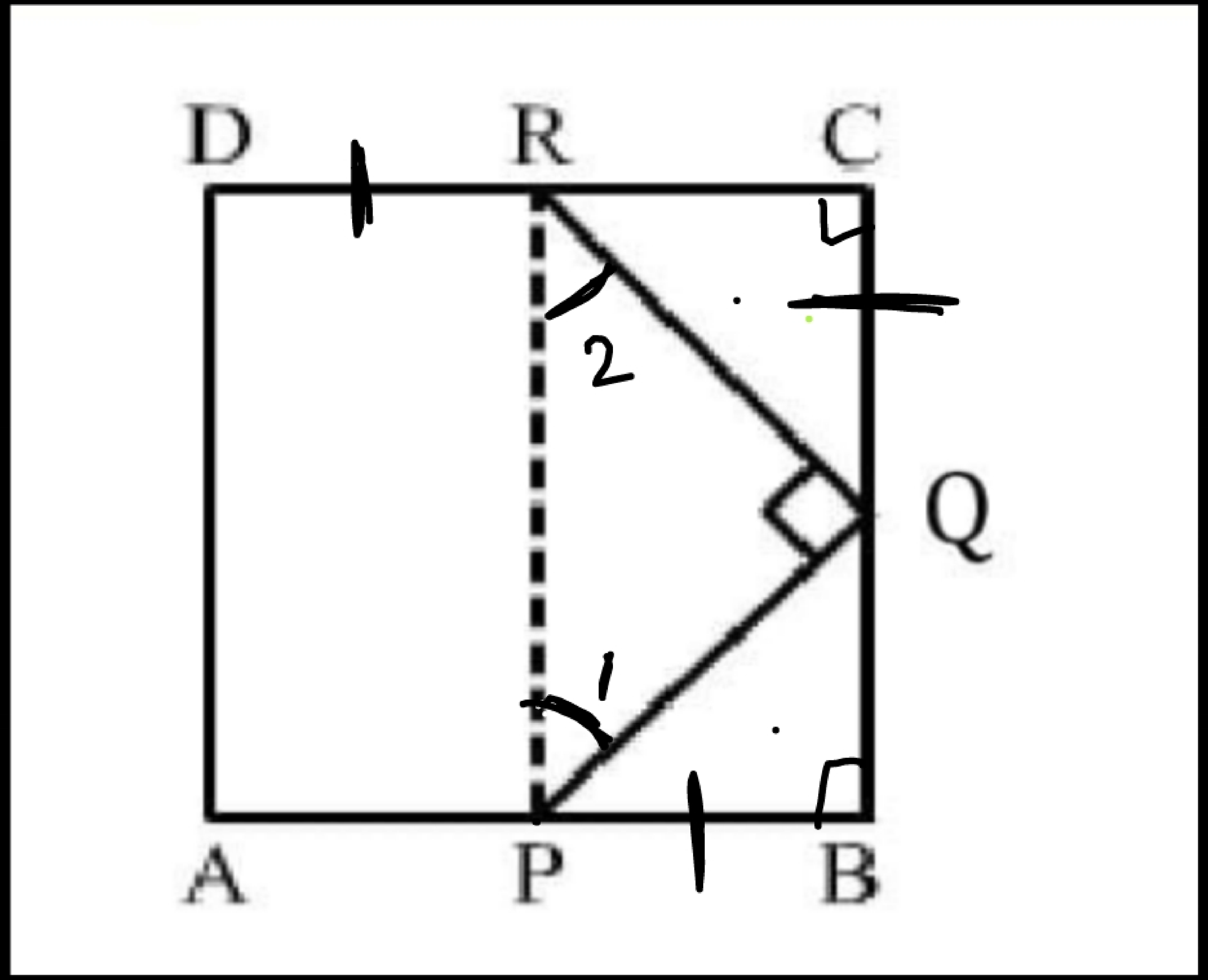
$\angle BAE = \angle DAE$ (Proved Above)

$\triangle AEB \cong \triangle ADE$ (by S.A.S criteria)

$\therefore BE = DE$

Proved

7. In the given figure, ABCD is a square and $\angle PQR = 90°$. If $PB = QC = DR$, prove that (i) $QB = RC$, (ii) $PQ = QR$, (iii) $\angle QPR = 45°$.



Given:- ABCD is a square, $\angle PQR = 90°$ and $PB = QC = DR$.

To Prove:- (i) $QB = RC$ (ii) $PQ = QR$ (iii) $\angle QPR = 45°$

Proof:- $BC = CD$ (sides of square)

$\Rightarrow BQ + \cancel{CQ} = \cancel{DR} + RC$ $\{\because CQ = DR\}$

$\Rightarrow BQ = RC$

In ΔRCQ and ΔQBP

$CR = QB$ (Proved Above)

$CQ = PB$ (given)

$\angle C = \angle B$ (each equal to 90°)

$\therefore \Delta RCQ \cong \Delta QBP$ {by S.A.S criteria}

Hence $RQ = PQ$ (by C.P.C.T)

In ΔQPR

$PQ = QR$

$\therefore \boxed{\angle 1 = \angle 2}$ { Angles opposite to the equal side of a Δ }

In ΔQPR

$\angle 1 + \angle 2 + \angle PQR = 180^\circ$

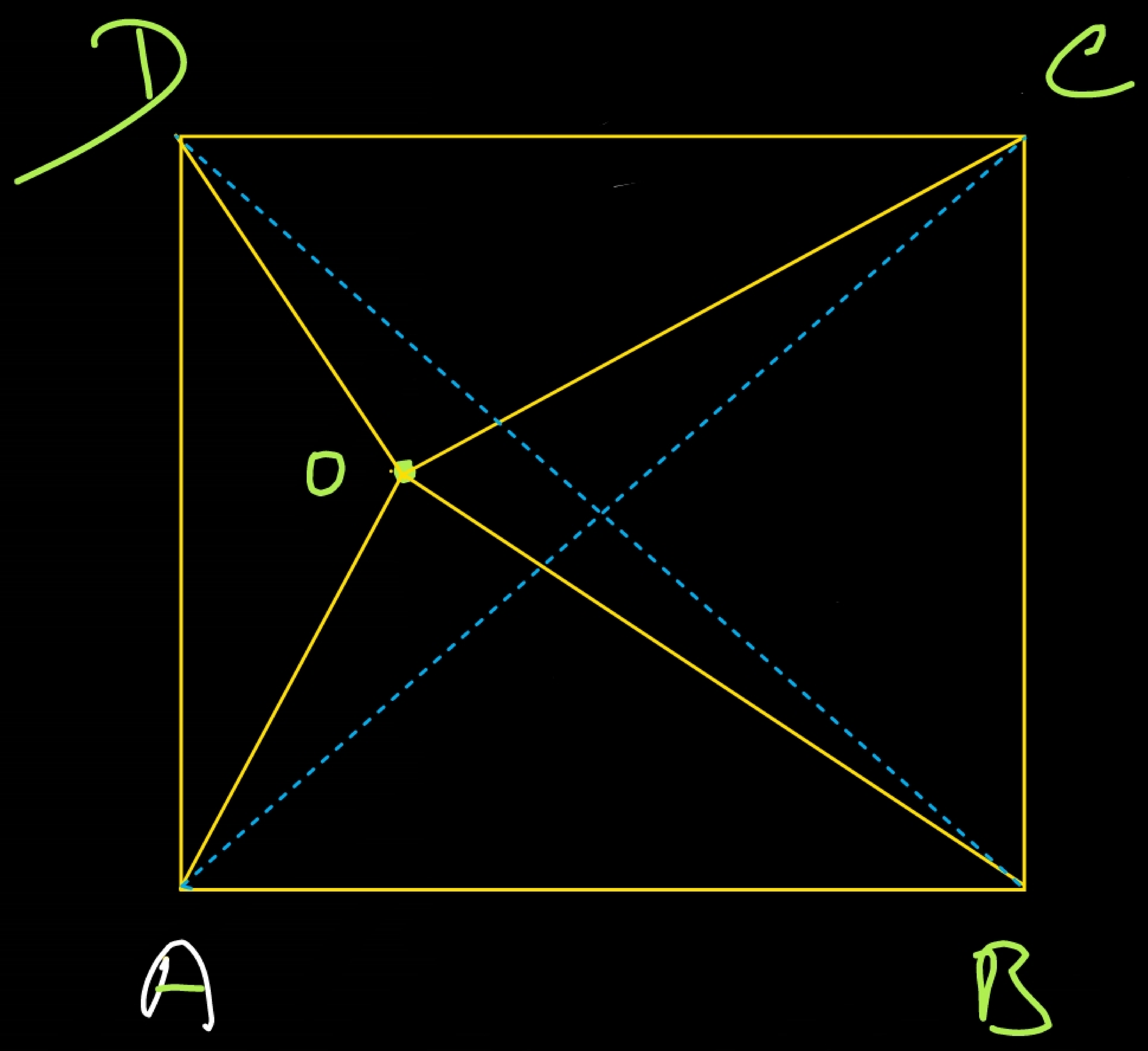
$\Rightarrow \angle 1 + \angle 1 + 90^\circ = 180$

$\Rightarrow 2\angle 1 = 90^\circ$

$\Rightarrow \angle 1 = 45^\circ$

$\therefore \boxed{\angle QPR = 45^\circ}$

8. If O is a point within a quadrilateral ABCD, show that OA + OB + OC + OD > AC + BD.



Given :- ABCD is a Quadrilateral, and O is a point inside the Quadrilateral

To Prove :- $OA + OB + OC + OD > AC + BD$

Proof :- In ΔAOC

$OA + OC > AC$ — (1) {the sum of two sides of a Δ is greater than 3rd side}

In ΔBOD

$OB + OD > BD$ — (2) (Same reason)

On adding eqⁿ 1 & 2

$$OA + OC + OB + OD > AC + BD$$

Hence $\boxed{OA + OB + OC + OD > AC + BD}$

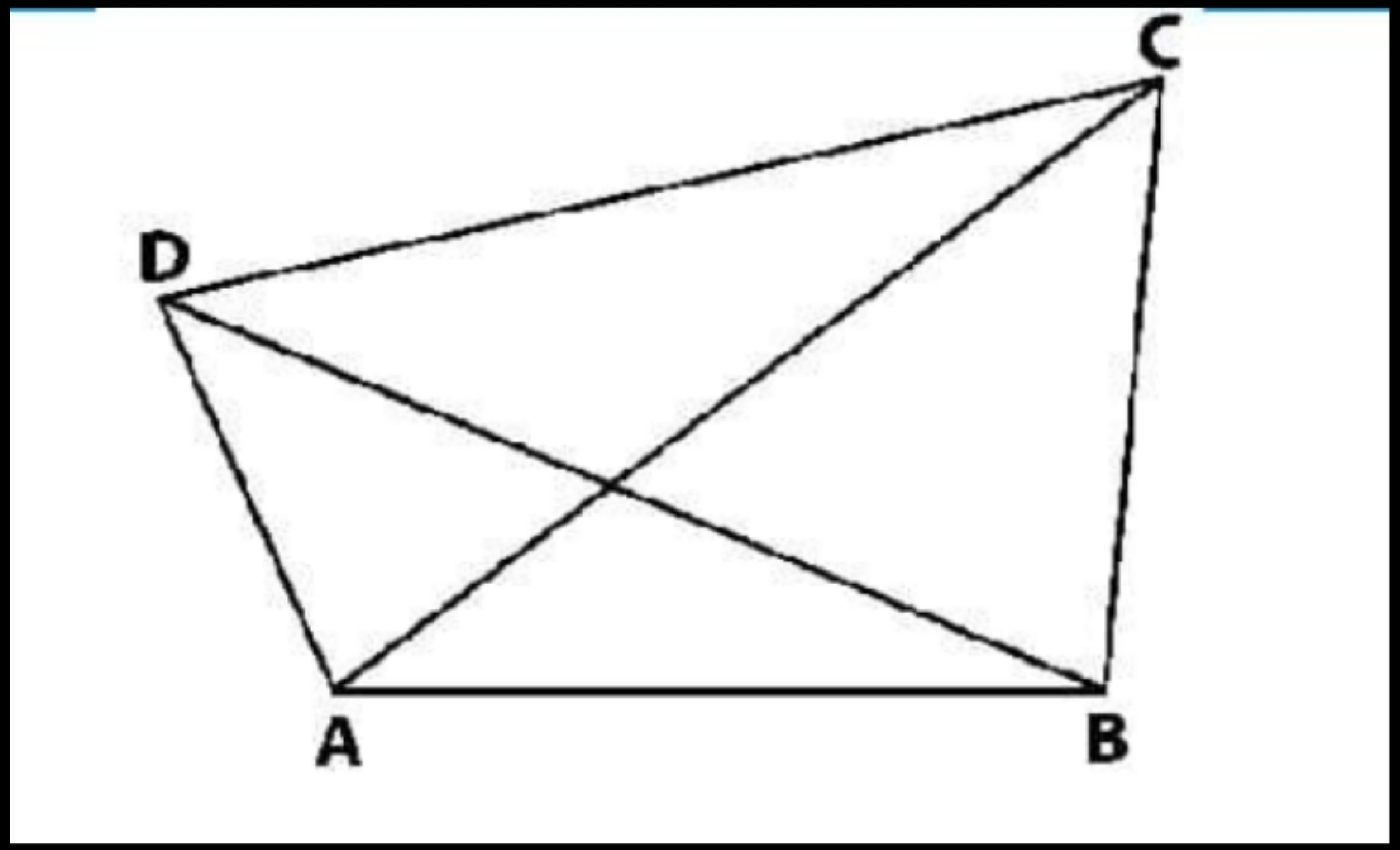
9. In the adjoining figure, ABCD is a quadrilateral and AC is one of its diagonals. Prove that

(i) AB + BC + CD + DA > 2AC

(ii) AB + BC + CD > DA

(iii) AB + BC + CD + DA > AC + BD.

Example 8 (i)



Given :- ABCD is a Quadrilateral

To Prove :- (1) AB + BC + CD + DA > 2AC

(2) AB + BC + CD > DA

(3) AB + BC + CD + DA > AC + BD

Proof:- In $\triangle ABC$

$AB + BC > AC$ — (1) {Sum of two side of a $\triangle$ is greater then 3rd side}

In $\triangle ACD$

$CD + DA > AC$ — (2) {Sum of two side of a $\triangle$ is greater then 3rd side}

On adding eqn (1) & (2)

$AB + BC + CD + DA > AC + AC$

$$\boxed{AB + BC + CD + DA > 2AC}$$

In $\triangle ABC$

$AB + BC > AC$ { Sum of two sides of a $\triangle$ is greater than the 3rd side }

On adding CD both side

$AB + BC + CD > AC + CD$ — (1)

In $\triangle ACD$

$AC + CD > DA$ — (2)

from eqn (1) & (2)

$AB + BC + CD > DA$

Proved.

10. Prove that the sum of all the angles of a quadrilateral is 360°.

Theorem I