Class 9th

RS AGGARWAL

Area OF Triangle And Quadrilateral

EXERCISE 14 A
EXAMPLES 1 TO 27

Areas of Triangles and Quadrilaterals

FORMULAE FOR AREA OF TRIANGLES

(i) Area of a Triangle = $\left(\frac{1}{2} \times \text{base} \times \text{height}\right)$ sq units.

(ii) **HERON'S FORMULA**

Let a, b, c be the sides of a triangle. Then,

semiperimeter, $s = \frac{1}{2}(a + b + c)$;

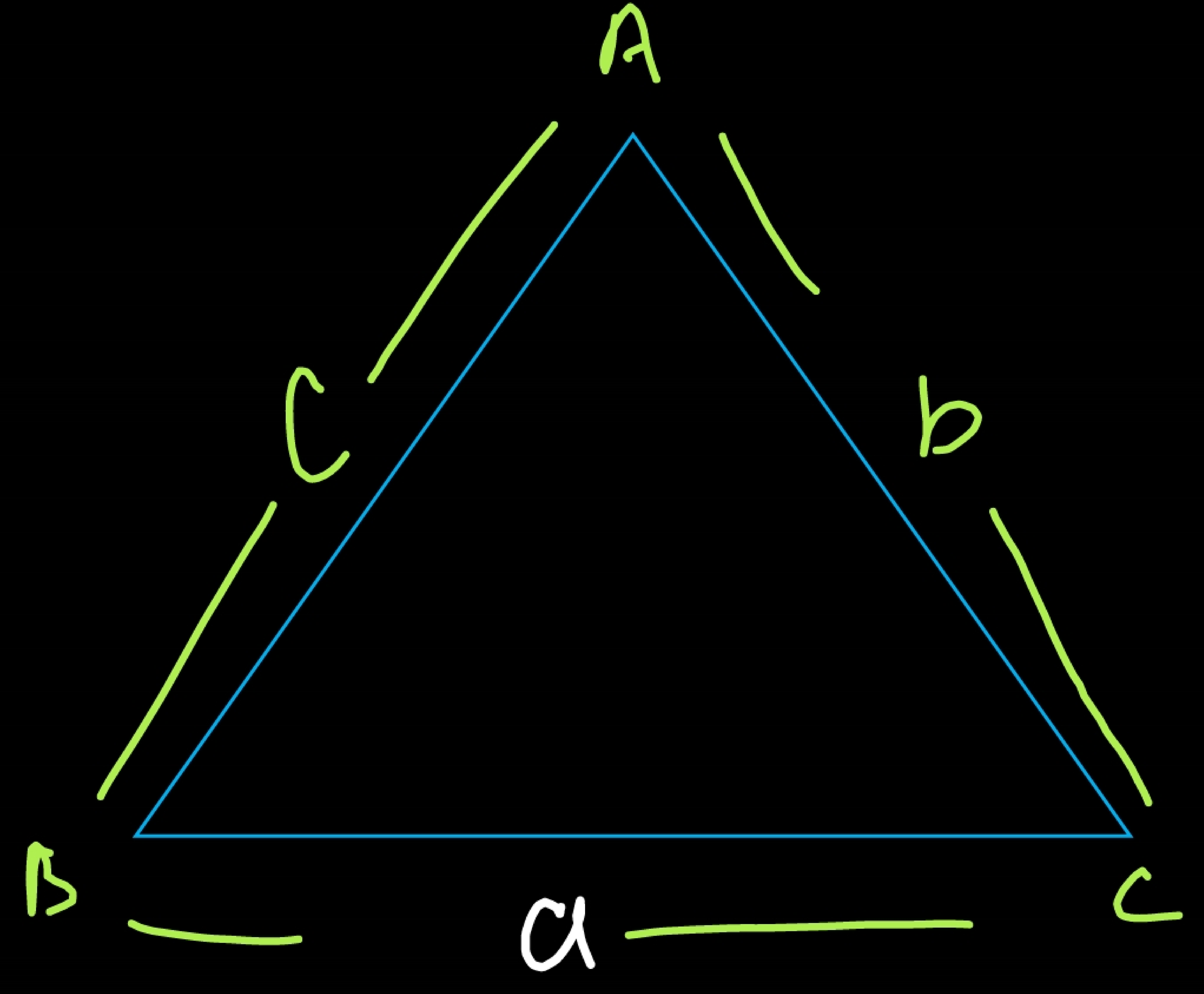
$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ sq units.}$$

(iii) Let each side of an equilateral triangle be a. Then,

$$\text{area} = \left(\frac{\sqrt{3}}{4} \times a^2\right) \text{ sq units, and height} = \left(\frac{\sqrt{3}}{2}a\right) \text{ units.}$$

(iv) Consider an isosceles triangle having base = b and each of equal sides = a. Then,

$$\text{area} = \left(\frac{b}{4} \times \sqrt{4a^2 - b^2}\right) \text{ sq units.}$$



$P = a + b + c$

$S = \frac{1}{2}(a + b + c)$

Example 1

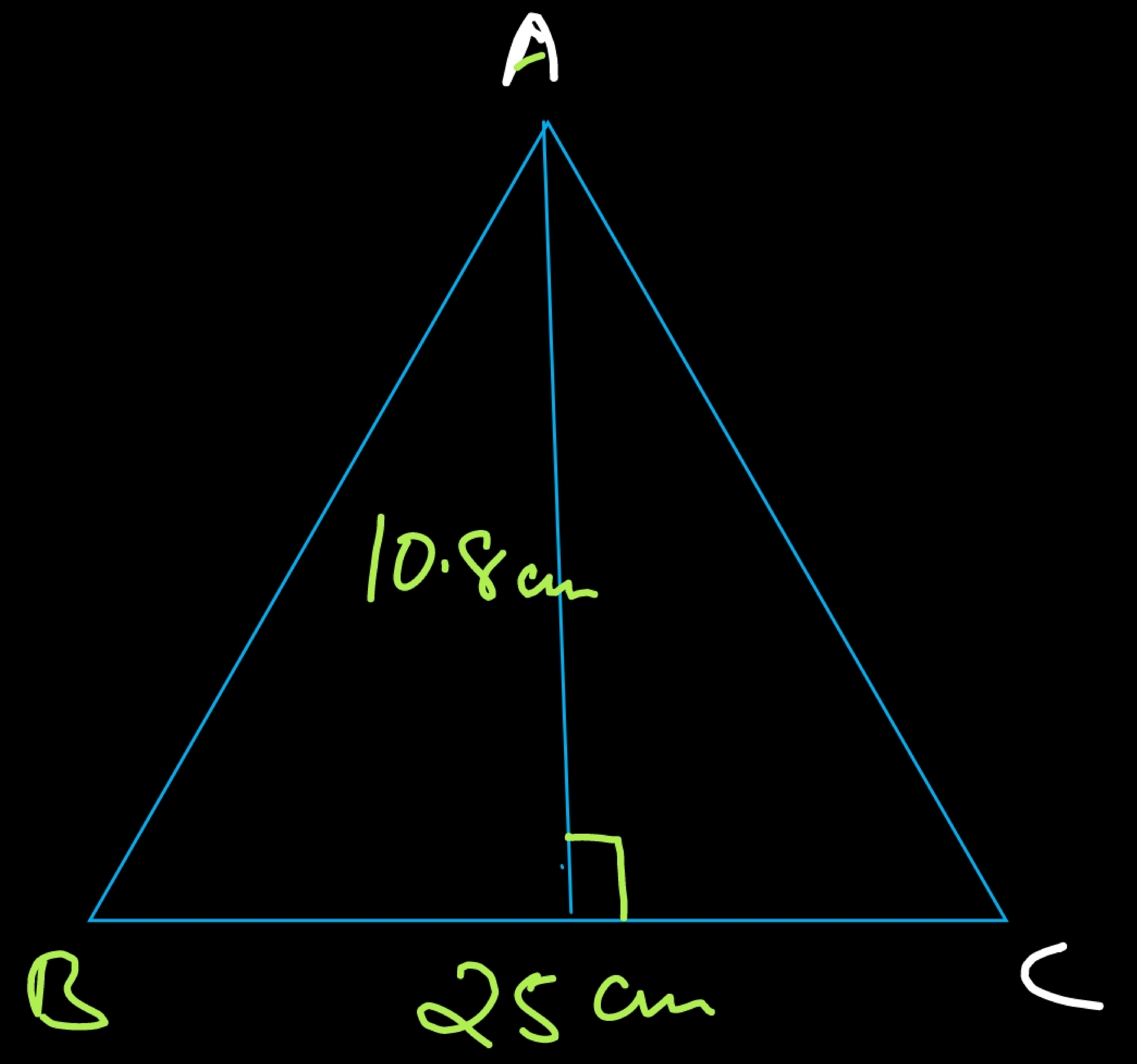
Find the area of a triangle whose base is 25 cm long and the correspondin height 10.8 cm.

$$\text{Ar. of } \Delta = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \left(\frac{1}{\cancel{2}} \times 25 \times \frac{\overset{54}{\cancel{108}}}{10}\right) cm^2$$

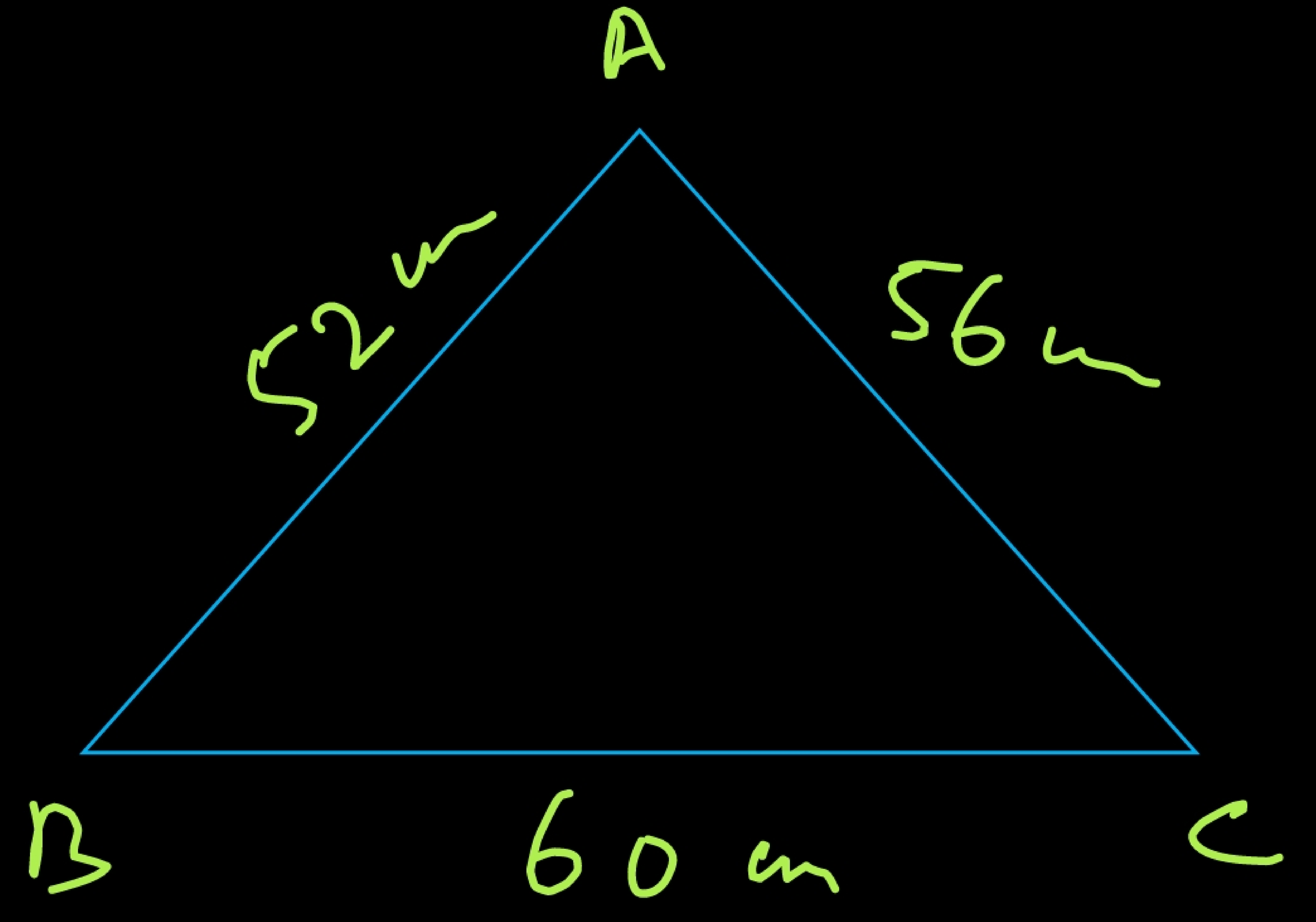
$$= \frac{135\cancel{0}}{\cancel{10}} cm^2$$

$$= 135\ cm^2 \text{ Ans}$$



EXAMPLE 2

Find the perimeter and area of a triangle whose sides are of lengths 52 cm, 56 cm and 60 cm respectively



Perimeter of $\Delta ABC = 60 + 56 + 52$

$= 168$ cm

Semiperimeter of $\Delta ABC = \frac{168}{2}$

$= 84$

As we know that from Heron's formula.

$$\text{Ar of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{84(84-60)(84-56)(84-52)}$$

$$= \sqrt{84 \times 24 \times 28 \times 32}$$

$$= \left(\sqrt{4 \times 7 \times 3 \times 4 \times 2 \times 3 \times 4 \times 7 \times 16 \times 2}\right) cm^2$$

$$= 4 \times 2 \times 4 \times 7 \times 3 \times 2 \ cm^2$$

$$= 64 \times 21 \ cm^2$$

$$= 1344 \ cm^2$$

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

EXAMPLE 3

The lengths of the sides of a triangle are in the ratio 3:4:5 and its perimeter is 144 cm. Find (i) the area of the triangle and (ii) the height corresponding to the longest side.

Let the ratio be x

$\therefore$ the side of the triangle $= 3x, 4x, 5x$

Perimeter of $\triangle = a+b+c$

$\Rightarrow 144 = 3x + 4x + 5x$

$\Rightarrow \cancel{144} = \cancel{12}x$

$\Rightarrow \boxed{x = 12}$

$\therefore$ Sides $= 3 \times 12$

$= 36$ cm

Now 2^{nd} side $= 4 \times 12$

$= 48$ cm

3^{rd} side $= 5 \times 12$

$= 60$ cm

$P = 144$ cm

$S = \frac{144}{2}$ cm

$\boxed{S = 72 \text{ cm}}$

(i) from Heron's formula

$$\text{Ar of } \Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{72(72-60)(72-36)(72-48)}$$

$$= \sqrt{36 \times 2 \times 12 \times 36 \times 24} \text{ cm}^2$$

$$= \sqrt{\underline{36} \times \underline{2} \times \underline{12} \times \underline{36} \times \underline{2} \times \underline{12}} \text{ cm}^2$$

$$= 36 \times 12 \times 2 \text{ cm}^2$$

$$= \underline{864} \text{ cm}^2$$

$\therefore$ Ar. of $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height}$

$\Rightarrow 864 = \frac{1}{\cancel{2}} \times \overset{30}{\cancel{60}} \times h$

$\Rightarrow \frac{864}{30} = h$

$\Rightarrow \boxed{28.8 \text{ cm} = h}$ ✓

EXAMPLE 4

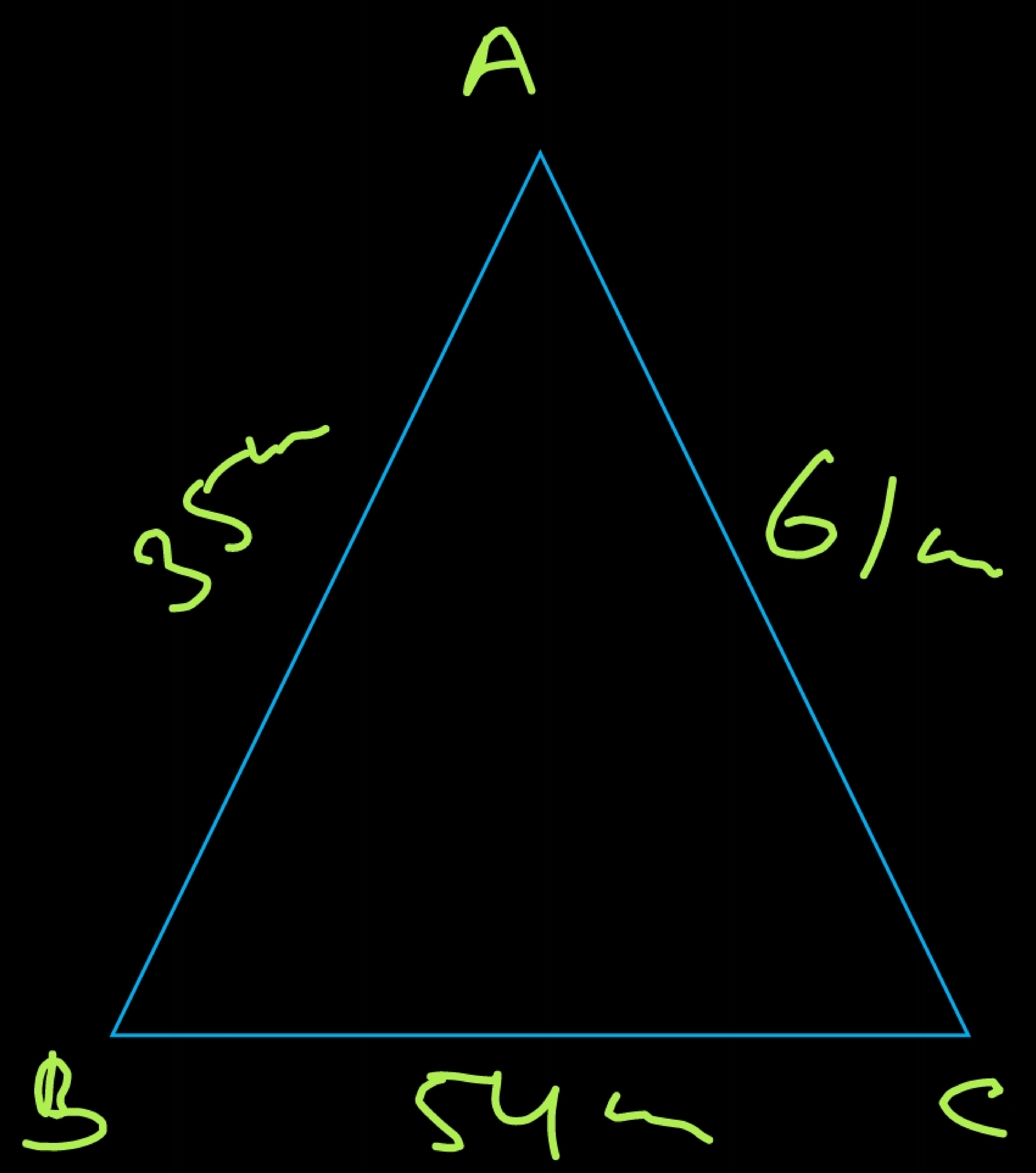
The sides of a triangle are 35 cm, 54 cm and 61 cm respectively. Find the length of its longest altitude.

$$\text{Semiperimeter } (S) = \frac{a+b+c}{2}$$

$$= \frac{35+54+61}{2}$$

$$= \frac{150}{2}$$

$$= 75 \text{ cm}$$



By Heron's formula

$$\text{Ar. of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{75(75-35)(75-54)(75-61)} \text{ cm}^2$$

$$= \sqrt{25 \times 3 \times 40 \times 21 \times 14} \text{ cm}^2$$

$$= \sqrt{25 \times \underline{3} \times 4 \times \underline{2} \times 5 \times \underline{3} \times \underline{7} \times \underline{2} \times \underline{7}} \text{ cm}^2$$

$$= 5 \times 3 \times 2 \times 2 \times 7 \times \sqrt{5} \text{ cm}^2$$

Ar. of $\triangle = 420\sqrt{5}\ cm^2$

Now,

Ar. of $\triangle = \frac{1}{2} \times$ Base $\times$ height

$\Rightarrow 420\sqrt{5} = \frac{1}{2} \times 35 \times h$

$$\Rightarrow \frac{\overset{12}{\cancel{420}} \times 2 \times \sqrt{5}}{\cancel{35}} = h$$

$\Rightarrow h = 24\sqrt{5}$ cm ✓

EXAMPLE 5

The perimeter of an equilateral triangle is 60 cm. Find its (i) area and (ii) height. (Given, √3 =1.732.)

Perimeter of an equilateral $\triangle = 3a$

$\Rightarrow \cancel{60} = \cancel{3}a$

$\Rightarrow \boxed{a = 20\ cm}$

Ar. of an equilateral $\triangle = \frac{\sqrt{3}}{4} \times side^2$

$= \left(\frac{\sqrt{3}}{\cancel{4}} \times \overset{5}{\cancel{20}} \times 20\right) cm^2$

$= 100\sqrt{3}\ cm^2$

$= 100 \times 1.732\ cm^2$

$= 173.2\ cm^2$

(n) Height of an equilateral $\Delta = \frac{\sqrt{3}}{2} \times a$

$= \frac{\sqrt{3}}{\cancel{2}} \times \overset{10}{\cancel{20}}\ cm$

$= 1.732 \times 10\ cm$

$= 17.32\ cm$

EXAMPLE 6

The height of an equilateral triangle is 6 cm. Find its area.

$$\text{Ar. of } \Delta = \frac{1}{2} \times a \times h$$

$$= \left(\frac{1}{\cancel{2}} \times a \times \cancel{6}^{3}\right) cm^2$$

$$= 3a \ cm^2$$

Now,

$$\text{Area of equilateral } \Delta = \frac{\sqrt{3}}{4} \times a^2$$

$$\Rightarrow 3\cancel{a} = \frac{\sqrt{3}}{4} \times \cancel{a} \times a$$

$$\Rightarrow a = \frac{4 \times 3}{\sqrt{3}}$$

$$\Rightarrow a = \frac{4 \times \sqrt{3} \times \cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$$

$$\Rightarrow \boxed{a = 4\sqrt{3} \ cm}$$

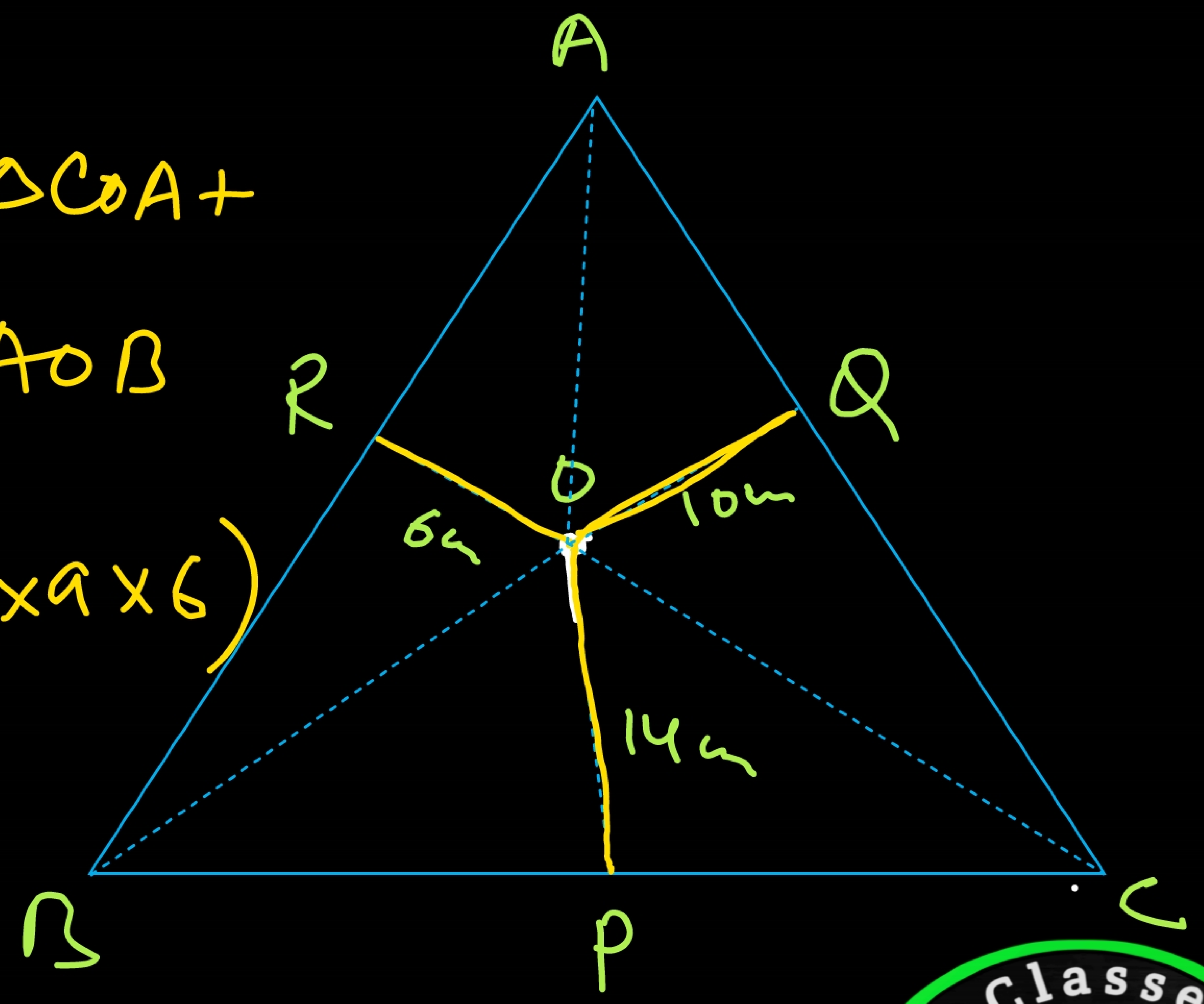
Ar. of equi. $\Delta = \frac{\sqrt{3}}{4} \times a^2$

$= \left(\frac{\sqrt{3}}{\cancel{4}} \times \cancel{4}\sqrt{3} \times 4\sqrt{3}\right) cm^2$

$= 12\sqrt{3}\ cm^2$

EXAMPLE 7

From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.



Ar. of equilateral Δ = Ar. of ΔBOC + Ar. of ΔCOA + Ar. of ΔAOB

$$= \left(\frac{1}{2} \times a \times 14 + \frac{1}{2} \times a \times 10 + \frac{1}{2} \times a \times 6\right)$$

$$= \frac{1}{2} a (14 + 10 + 6) \text{ cm}^2$$

$$= \frac{1}{2} a \times 30$$

$$= 15a \text{ cm}^2$$

Ar. of equilateral $\Delta = \frac{\sqrt{3}}{4} \times a^2$

$$\Rightarrow 15\cancel{a} = \frac{\sqrt{3}}{4} \times \cancel{a} \times a$$

$$\Rightarrow \frac{15 \times 4}{\sqrt{3}} = a$$

$$\Rightarrow \frac{3 \times 5 \times 4}{\sqrt{3}} = a$$

$$\Rightarrow \frac{\cancel{\sqrt{3}} \times \sqrt{3} \times 5 \times 4}{\cancel{\sqrt{3}}} = a$$

$$\Rightarrow \boxed{20\sqrt{3}\ cm = a}$$

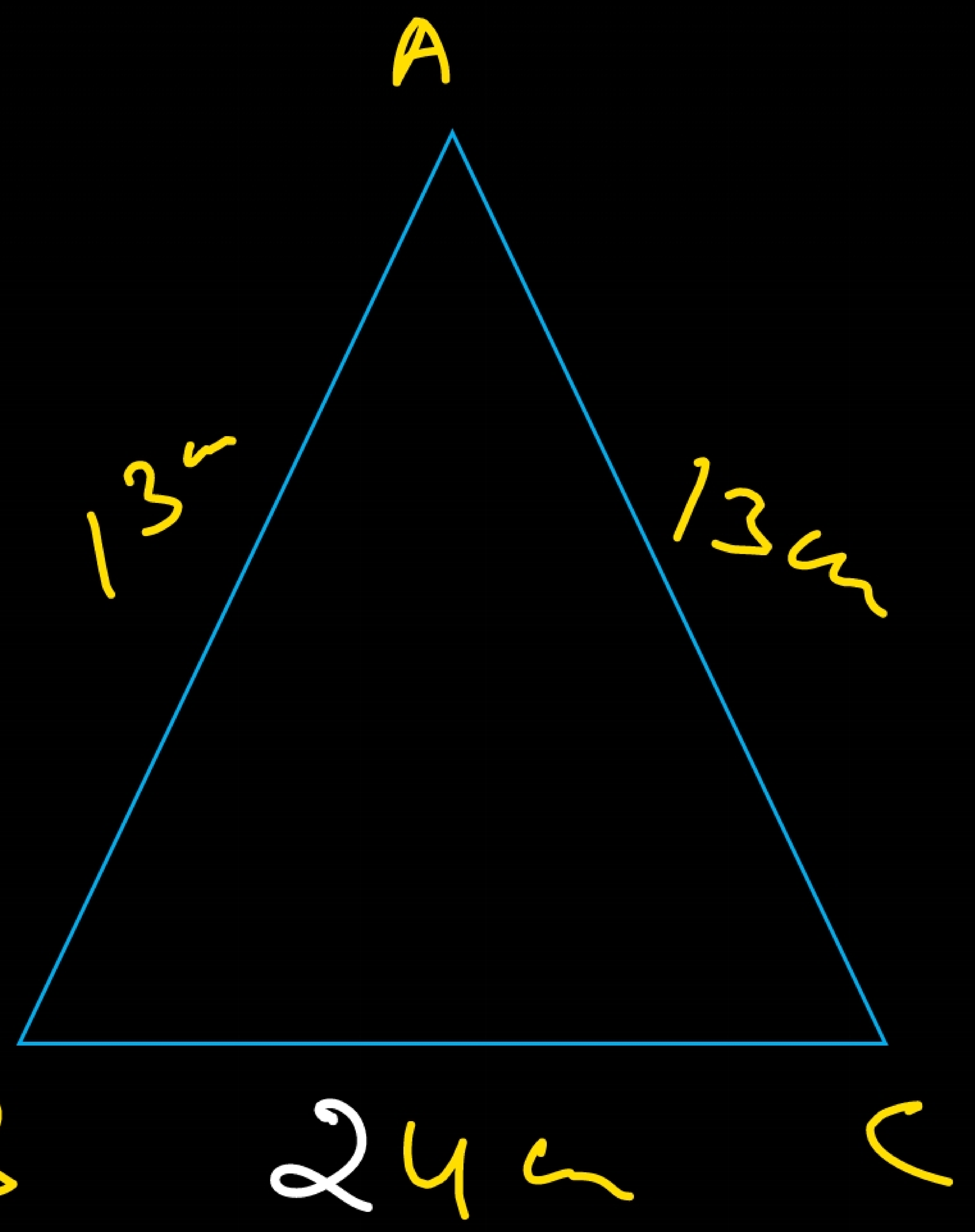
<u>Now</u> Ar. of equilateral $\Delta = 15a \text{ cm}^2$

$= 15 \times 20\sqrt{3} \text{ cm}^2$

$= 300\sqrt{3} \text{ cm}^2$

EXAMPLE 8

Find the area of an isosceles triangle each of whose equal sides is 13 cm and whose base is 24 cm.



$$\text{Ar. of isosceles } \Delta = \frac{b}{4}\sqrt{4a^2-b^2}$$

$$= \frac{\cancel{24}}{4}\sqrt{4\times 13^2-24^2}\ \text{cm}^2$$

$$= 6\sqrt{4\times 169-576}\ \text{cm}^2$$

$$= 6\sqrt{676-576}\ \text{cm}^2$$

$$= 6\sqrt{100}\ \text{cm}^2$$

$$= 6\times 10\ \text{cm}^2$$

$$= 60\ \text{cm}^2$$

EXAMPLE 9

The base of an isosceles triangle measures 24 cm and its area is $192 cm^2$ Find its perimeter.

Ar. of an isosceles $\Delta = \frac{b}{4}\sqrt{4a^2 - b^2}$

$\Rightarrow 192 = \frac{\cancel{24}^{6}}{\cancel{4}}\sqrt{4a^2 - 24^2}$

$\Rightarrow \left(\frac{\cancel{192}^{32}}{\cancel{6}}\right)^2 = 4a^2 - 576$

$= 32^2 + 576 = 4a^2$

$\Rightarrow \frac{1024 + 576}{4} = a^2$

$\Rightarrow \frac{\cancel{1600}}{\cancel{4}} = a^2$

$\Rightarrow \sqrt{400} = a$

$\Rightarrow \boxed{a = 20\,m}$

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$$\text{Perimeter of } \Delta = 2a + b$$

$$= 2 \times 20 + 24 \text{ cm}$$

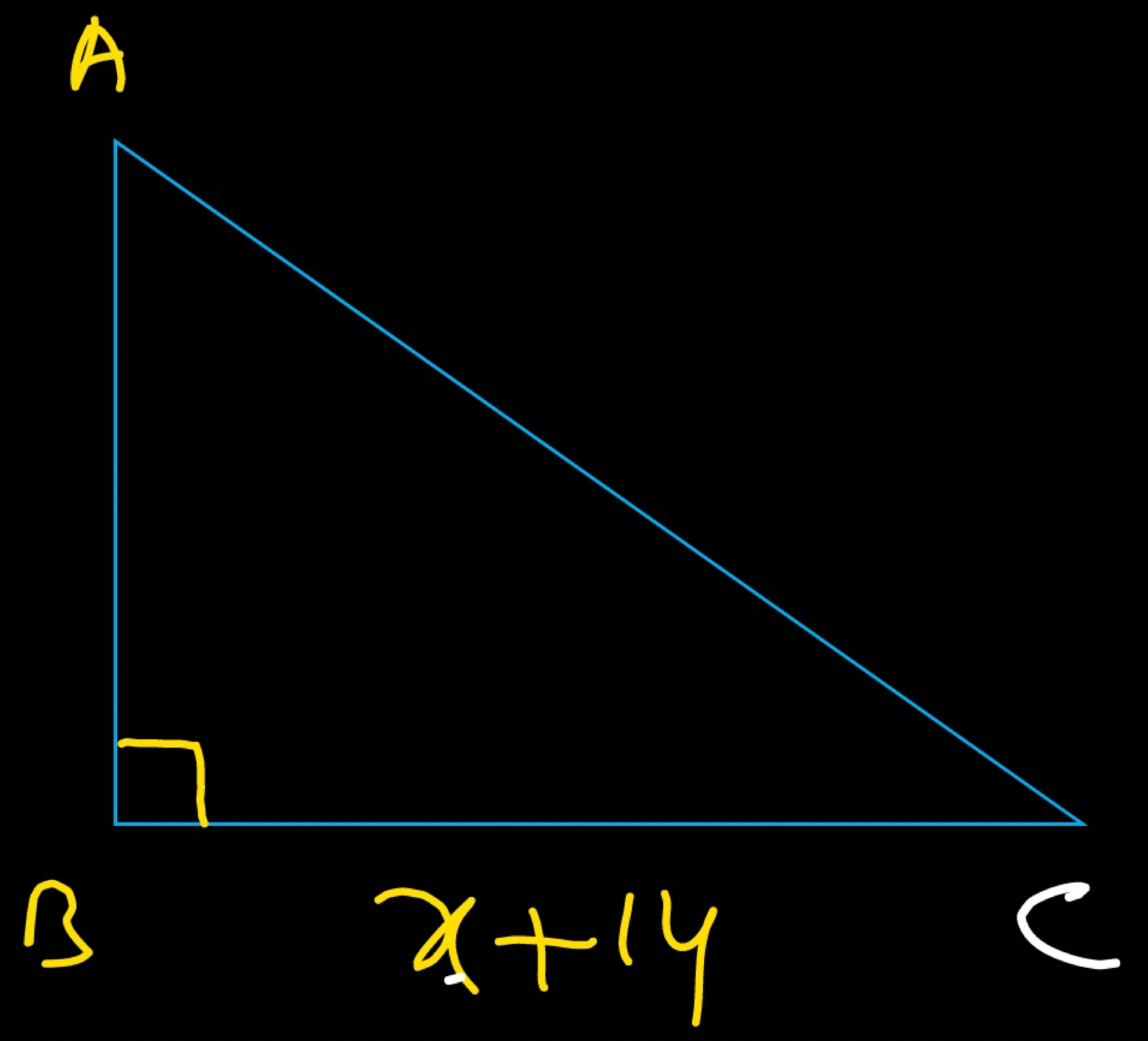
$$= 40 + 24 \text{ cm}$$

$$= 64 \text{ cm}$$

EXAMPLE 10

The difference between the sides at right angles in a right-angled triangle is 14 cm. The area of the triangle is 120cm^2 Calculate the perimeter of the triangle.

Let one side of right angled triangle be x



Ar. of right angled $\Delta = \frac{1}{2} \times$ Base $\times$ height

x.

$$\Rightarrow 120 = \frac{1}{2} \times (x+14)\, x$$

$$\Rightarrow 240 = x^2 + 14x$$

$$\Rightarrow x^2 + 14x - 240 = 0$$

$$\Rightarrow x^2 + 14x - 240 = 0$$

$$-240x^2$$

$$\Rightarrow x^2 + 24x - 10x - 240 = 0$$

$$\Rightarrow x(x+24) - 10(x+24) = 0$$

$$\Rightarrow (x+24)(x-10) = 0$$

2	240
2	120
2	60
2	30
3	15
	5

$$\Rightarrow x + 24 = 0 \quad \text{or} \quad x - 10 = 0$$

$$\Rightarrow x = -24\,\text{cm}\ \times \quad \text{or} \quad x = 10\,\text{cm}$$

$\therefore$ The sides of equilateral Δ are $= 10\,\text{cm},\ 24,$

By Pythagarous theorem, we know that

$$\Rightarrow P^2 + B^2 = H^2$$

$$\Rightarrow 10^2 + 24^2 = H^2$$

$$\Rightarrow 100 + 576 = H^2$$

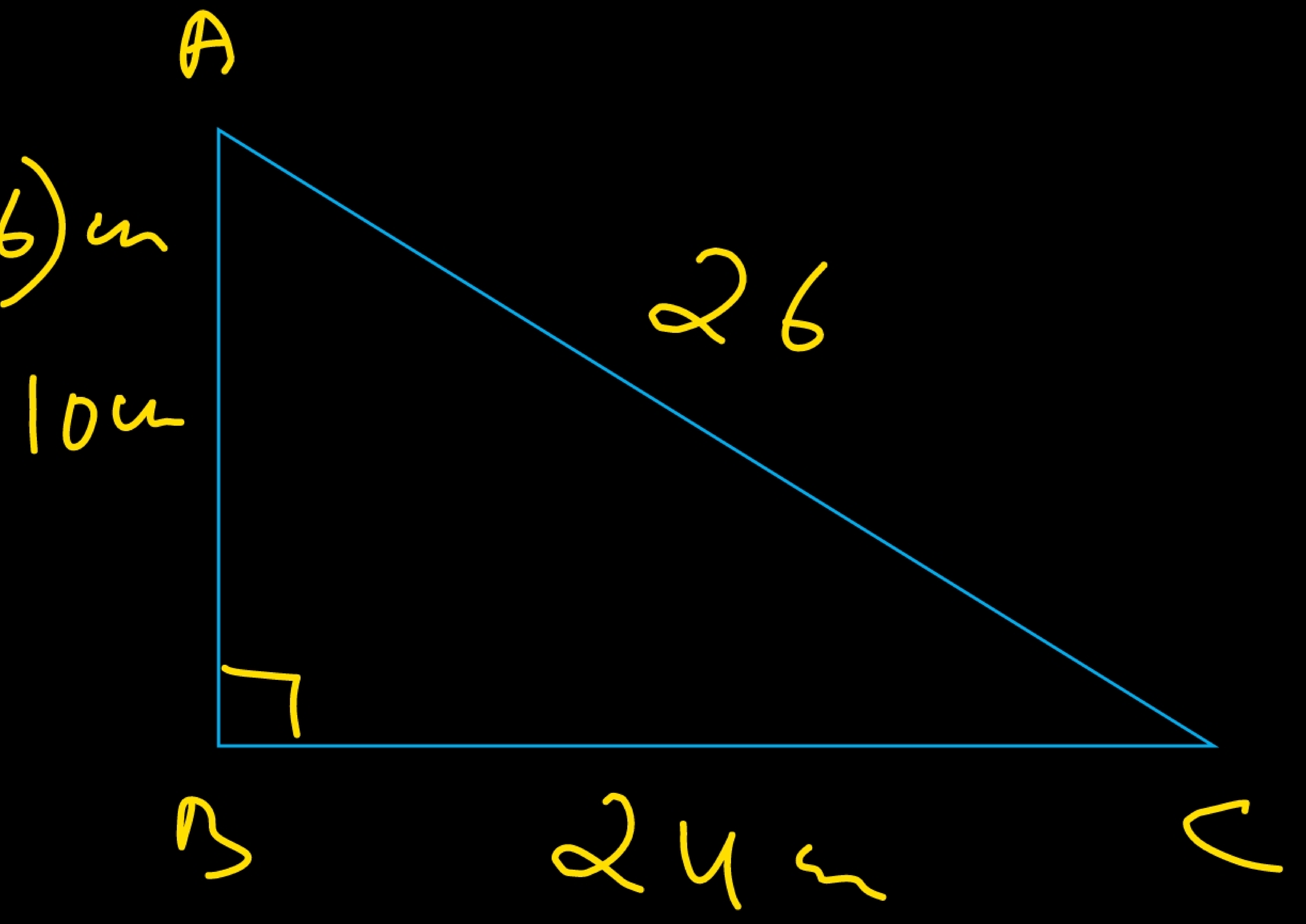
$$\Rightarrow \sqrt{676} = H$$

$$\Rightarrow \boxed{26 = H}$$

$\therefore$ Perimeter of

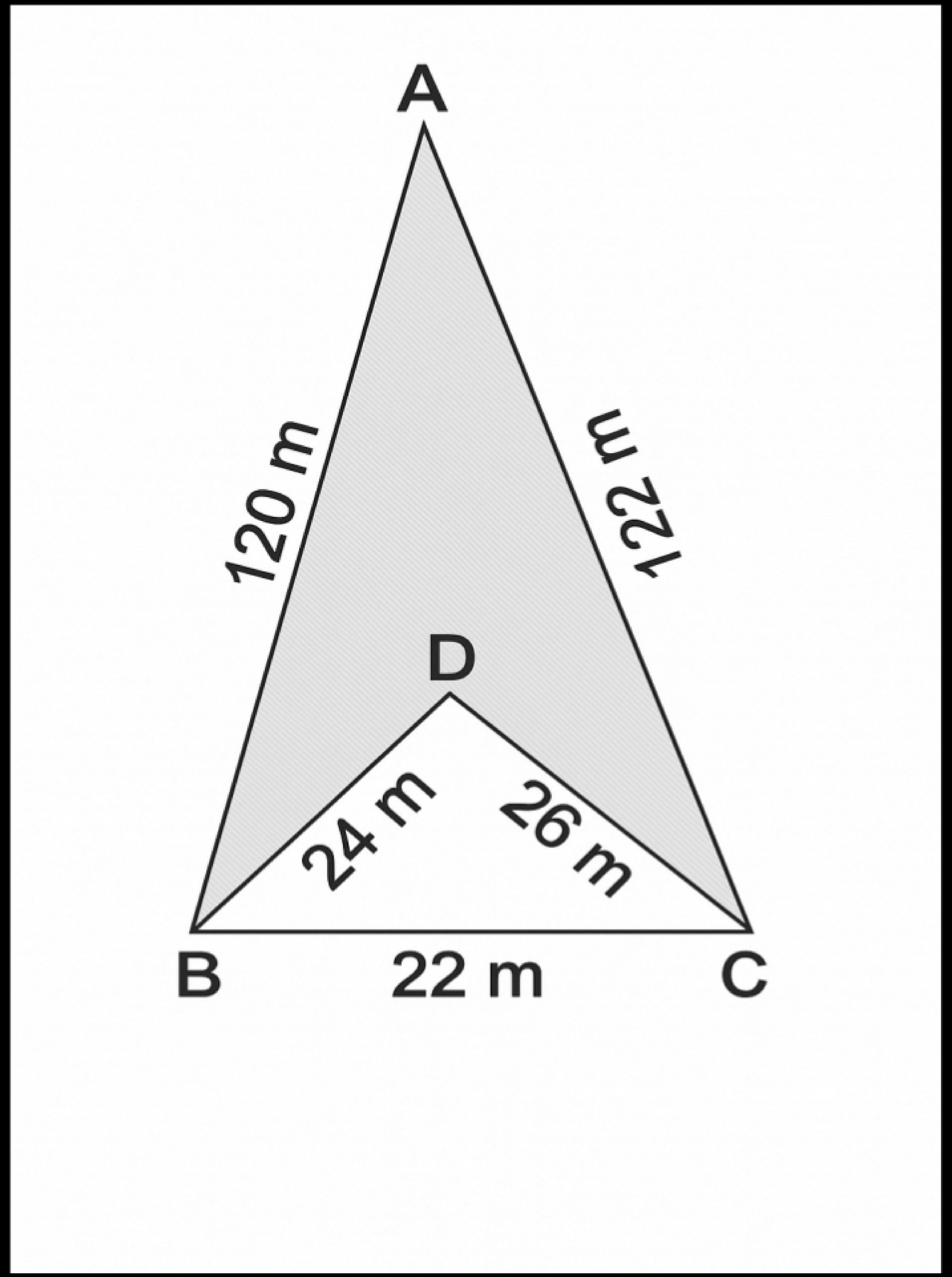
$$\triangle = (10 + 24 + 26)\text{ cm}$$

$$= \underline{\underline{60\text{ cm}}}$$



EXAMPLE 11

Calculate the area of the shaded region in the given figure.



Now, Area of shaded region = Ar. of $\triangle ABC$ − Ar. of $\triangle BCD$

S. of $\triangle ABC = \frac{120+122+22}{2}$ cm

$= \frac{264}{2}$ cm

$= 132$ cm

S. of $\triangle BCD = \frac{22+26+24}{2}$

$= \frac{72}{2}$ cm

$= 36$ cm

$\therefore$ Ar of shaded region = Ar. of $\triangle ABC$ − Ar. of $\triangle BCD$

$$= \left[\sqrt{132(132-22)(132-122)(132-120)} - \sqrt{36(36-22)(36-24)(36-26)}\right]$$

$$= \left[\sqrt{132 \times 110 \times 10 \times 12} - \sqrt{36 \times 14 \times 12 \times 10}\right] \text{ cm}^2$$

$$= \left[\sqrt{2\times2\times3\times11\times2\times5\times11\times2\times5\times2\times2\times3} - \sqrt{36\times2\times7\times4\times3\times2\times5}\right] \text{ cm}^2$$

$$= 2\times3\times11\times2\times5\times2 - 6\times2\times2\sqrt{105}$$

$$= 1320 - 24\times10.24$$

$$= 1320 - 245.76$$

$$= 1074.24 \text{ cm}^2$$

$$\begin{array}{r|l|l}
+1 & \overline{1\,05} & 10.24 \\
1 & 1 & \\
\hline
202 & 20500 & \\
\times 2 & 404 & \\
\hline
2044 & 09600 & \\
4 & 8176 & \\
\hline
\end{array}$$

EXAMPLE 12

Find the area of the quadrilateral ABCD in which AB = 9cm, BC = 40cm, CD = 28cm, DA = 15 cm and < ABC =90⁰

By Pythagorous theorem.

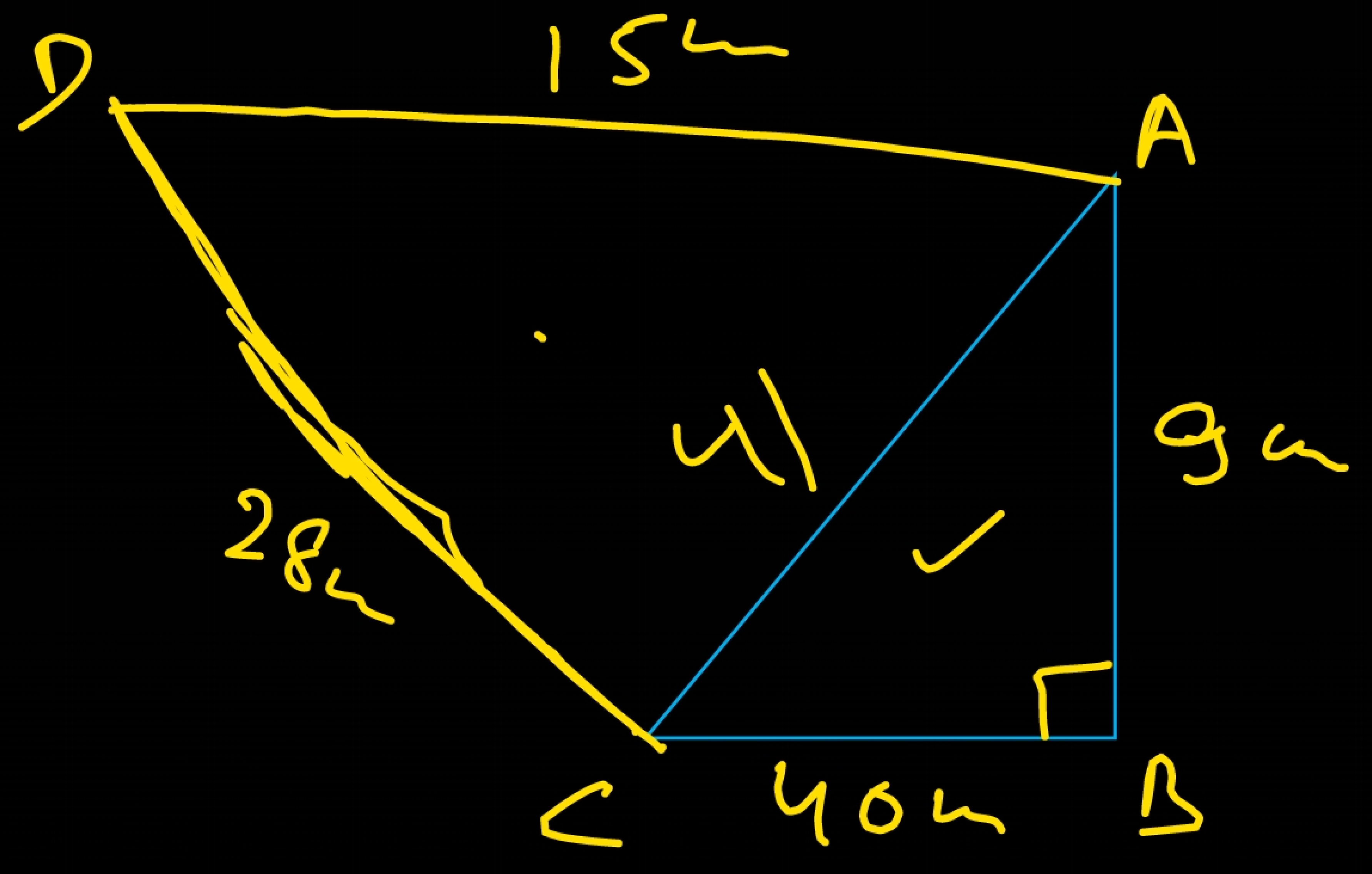
$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 40^2 + 9^2$$

$$\Rightarrow AC = \sqrt{1600 + 81}$$

$$\Rightarrow AC = \sqrt{1681}$$

$$\Rightarrow \boxed{AC = 41 \text{ cm}}$$



$$S \text{ of } \triangle ACD = \frac{28+15+41}{2}$$

$$= \frac{\cancel{84}}{\cancel{2}}$$

$$= 42$$

$$\text{Ar. of Quad. } ABCD = \text{Ar of } \triangle ABC + \text{Ar. of } \triangle ACD$$

$$= \frac{1}{\cancel{2}} \times \overset{20}{\cancel{40}} \times 9 + \sqrt{42(42-28)(42-15)(42-41)}$$

$$= 180 + \sqrt{\underline{3} \times \underline{7} \times \underline{2} \times 2 \times 7 \times 3 \times 9 \times 1}$$

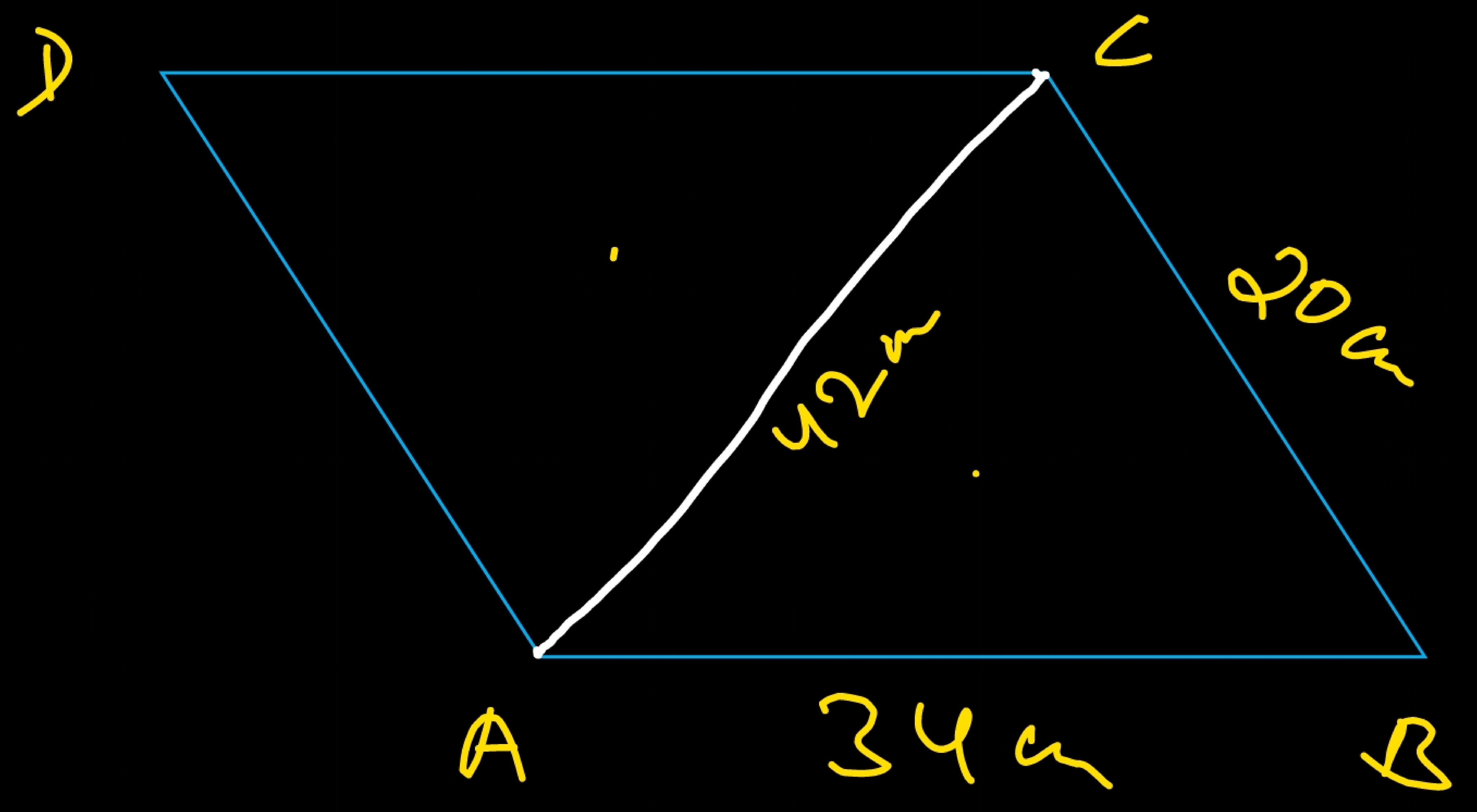
$$= (180 + 3 \times 7 \times 2 \times 3) \text{ cm}^2$$

$$= (180 + 126) \text{ cm}^2$$

$$= 306 \text{ cm}^2$$

EXAMPLE 13

The adjacent sides of a parallelogram ABCD are AB = 34 cm, BC = 20 cm and diagonal AC = 42 cm. Find the area of the parallelogram.



$$\text{Semi. P.} = \frac{34 + 42 + 20}{2}$$

$$= \frac{96}{2} \text{ cm}$$

$$= 48 \text{ cm}$$

$$\text{Ar. of } \| \text{gm } ABCD = 2 \times \text{Ar. of } \triangle ABC$$

$$= 2 \times \sqrt{48(48-34)(48-20)(48-42)}$$

$$= 2 \times \sqrt{48 \times 14 \times 28 \times 6}$$

$\therefore$ Ar. of ||gm ABCD $= \left(2 \times \sqrt{16 \times 3 \times 14 \times 14 \times 2 \times 2 \times 3}\right) cm^2$

$= 2 \times 4 \times 3 \times 14 \times 2 \; cm^2$

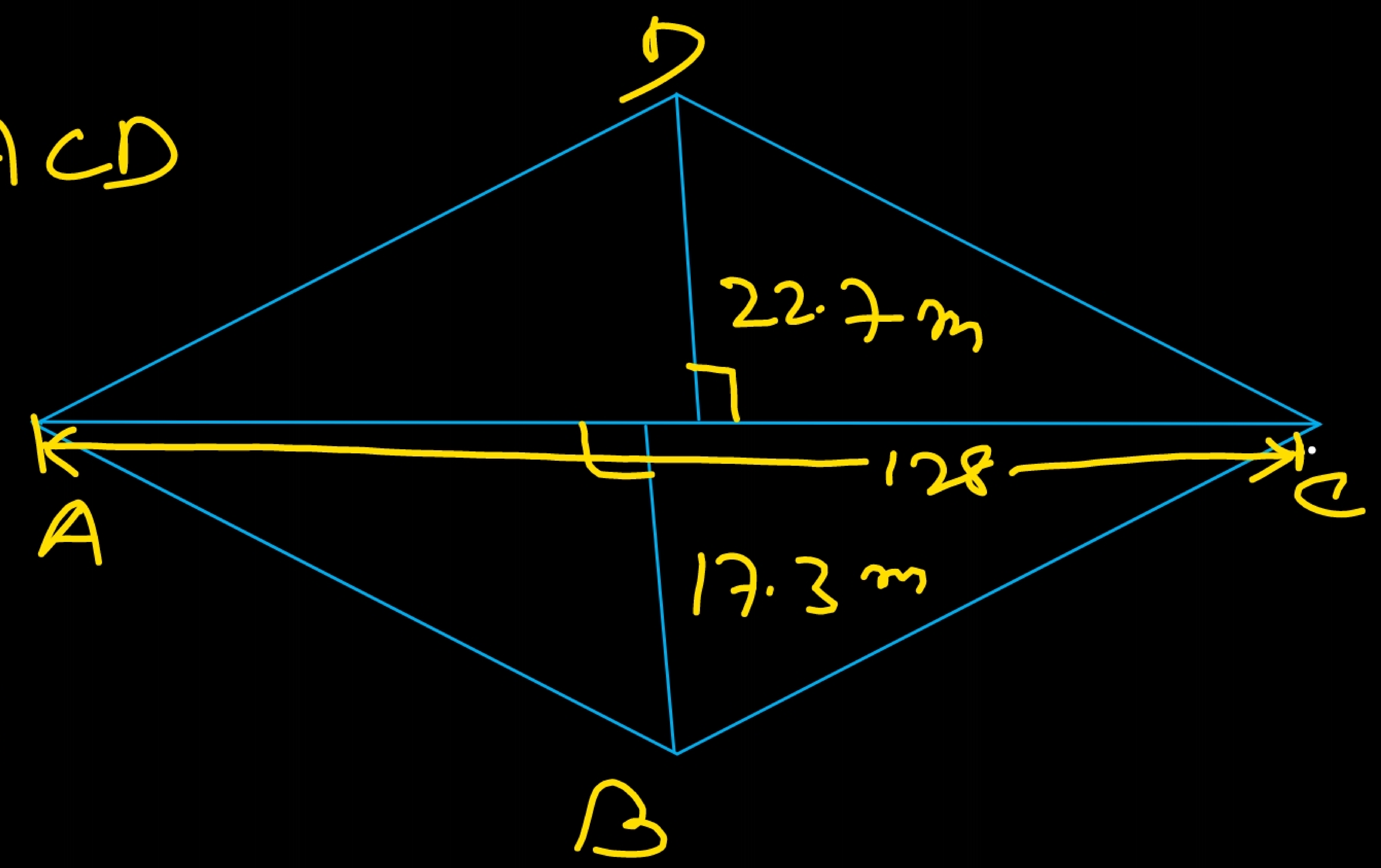
$= 48 \times 14 \; cm^2$

$\boxed{\text{Ar. of ||gm ABCD} = 672 \; cm^2}$

EXAMPLE 14

In a four-sided field, the length of the longer diagonal is 128 m. The lengths of the perpendiculars from the opposite vertices upon this diagonal are 22.7 m and 17.3 m. Find the area of the field.

Ar. of Quad. ABCD = Ar. of ΔABC + Ar. of ΔACD



$$= \frac{1}{2} \times 128 \times 17.3 + \frac{1}{2} \times 128 \times 22.7$$

$$= \frac{1}{\cancel{2}} \times \overset{64}{\cancel{128}} (17.3 + 22.7)$$

$$= 64 \times 40 \text{ cm}^2$$

$$= 2560 \text{ cm}^2$$

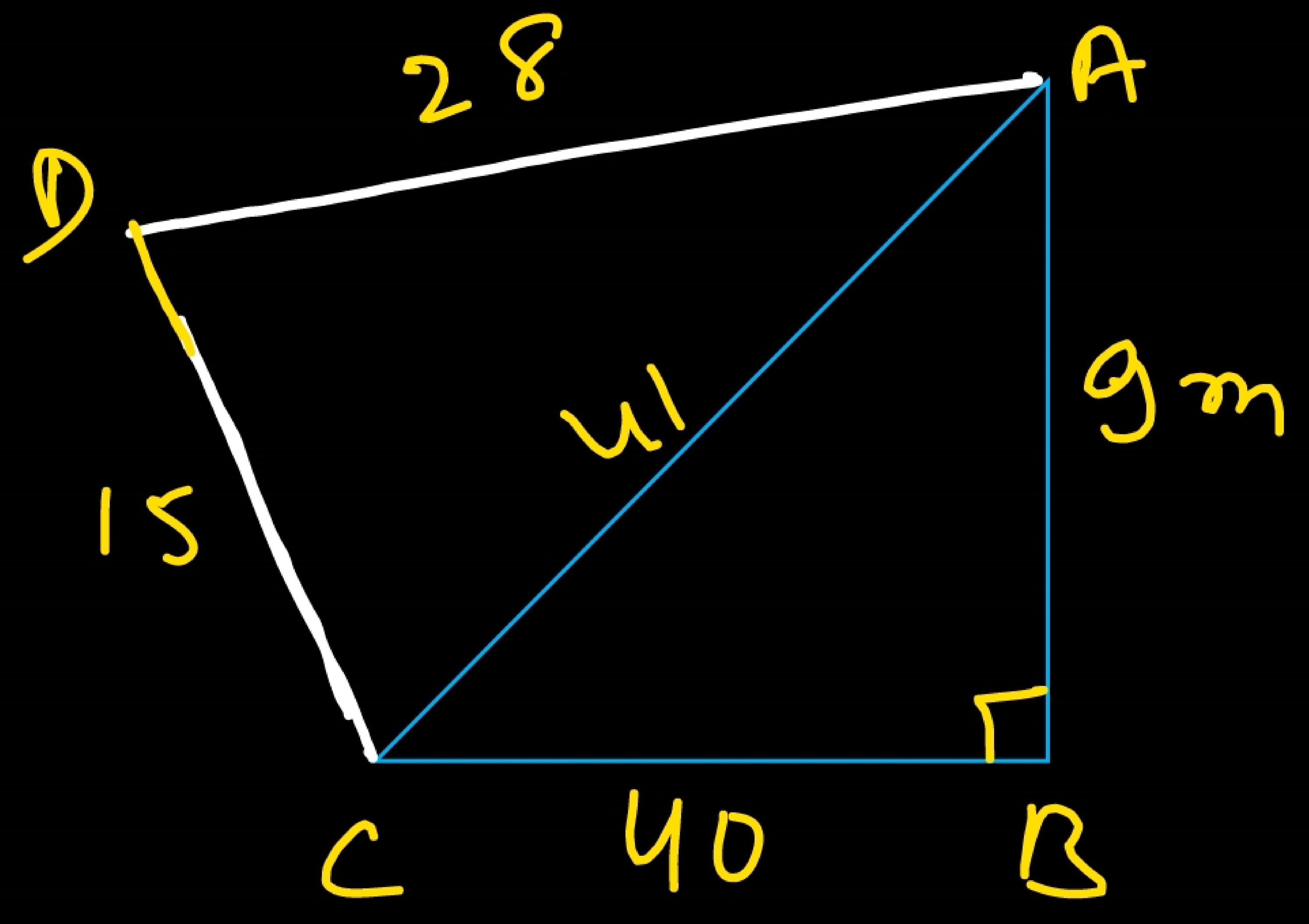
EXAMPLE 15

Find the area of the quadrilateral ABCD in which AB = 9 m, BC = 40 m, $\angle ABC = 90°$, CD = 15 m and AD = 28 m.

By Pythagorous theorem

$$AC = \sqrt{40^2 + 9^2}$$

$$\boxed{AC = 41 \text{ cm}}$$



$$S.\ \text{Perimeter of } \triangle ACD = \frac{15 + 41 + 28}{2}$$

$$= \frac{\cancel{84}}{\cancel{2}}$$

$$= 42$$

Ar. of Quad. ABCD = Ar. of $\triangle ABC$ + Ar. of $\triangle CAD$

$$= \frac{1}{\cancel{2}} \times \overset{20}{\cancel{40}} \times 9 + \sqrt{42(42-41)(42-28)(42-15)}$$

$$= 180 + \sqrt{42 \times 1 \times 14 \times 27}$$

$$= 180 + \sqrt{14 \times 3 \times 14 \times 3 \times 9}$$

$$= (180 + 14 \times 3 \times 3) \text{ cm}^2$$

$$= (180 + 126) \text{ cm}^2$$

$$= 306 \text{ cm}^2$$

EXAMPLE 16

A piece of land is in the shape of a rhombus whose perimeter is 400 m and one of its diagonals is 160 m. Find the area of the land.

Perimeter of rhombus = 400 m

$\Rightarrow \cancel{4}a = \cancel{400}$

$\Rightarrow \boxed{a = 100}$ m

In right angled triangle AOB

$\Rightarrow AB^2 = AO^2 + OB^2$

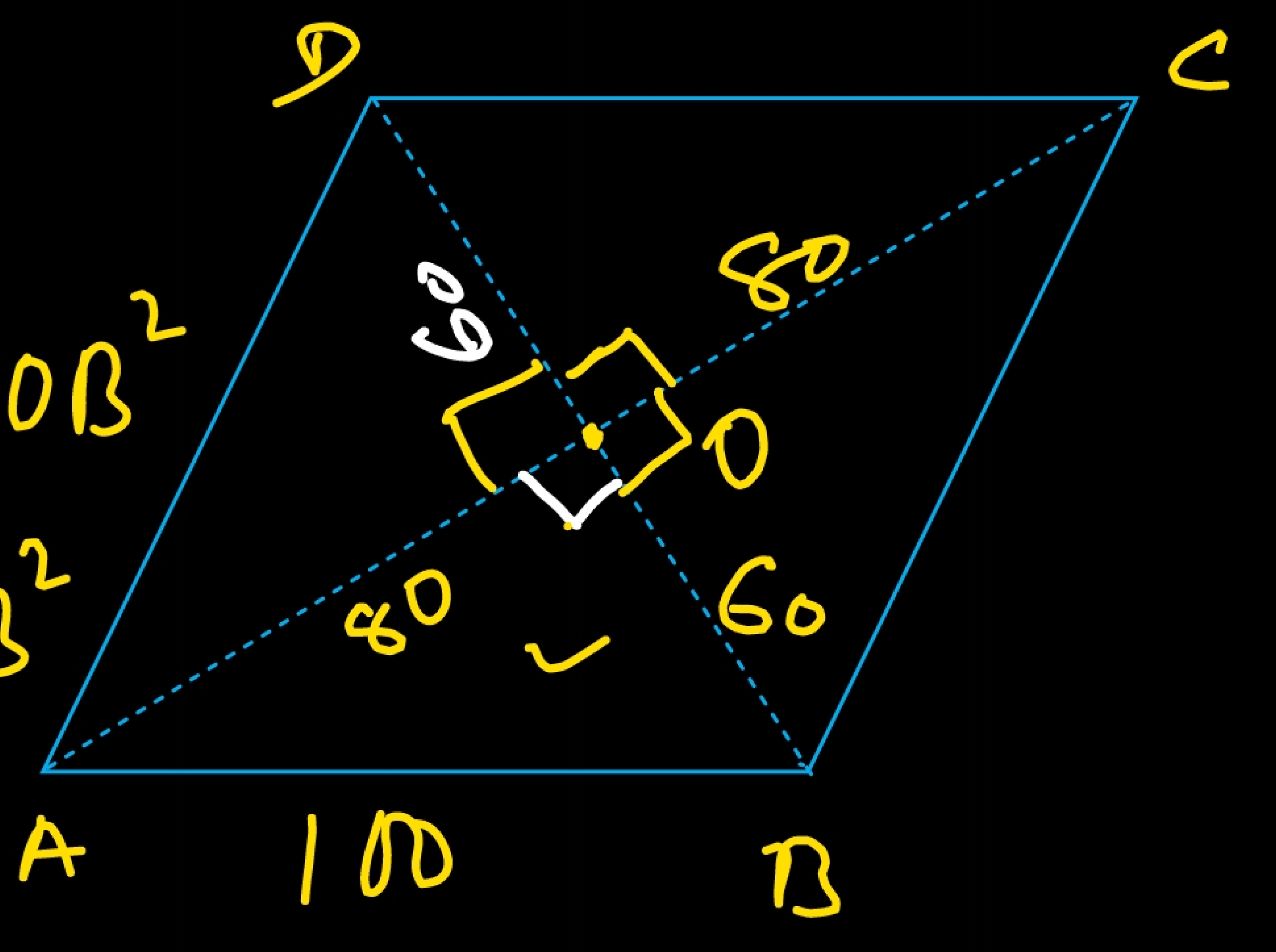
$\Rightarrow 100^2 = (80)^2 + OB^2$

$\Rightarrow 10000 = 6400 + OB^2$

$\Rightarrow 10000 - 6400 = OB^2$

$\Rightarrow 3600 = OB^2$

$\Rightarrow \boxed{60 = OB}$



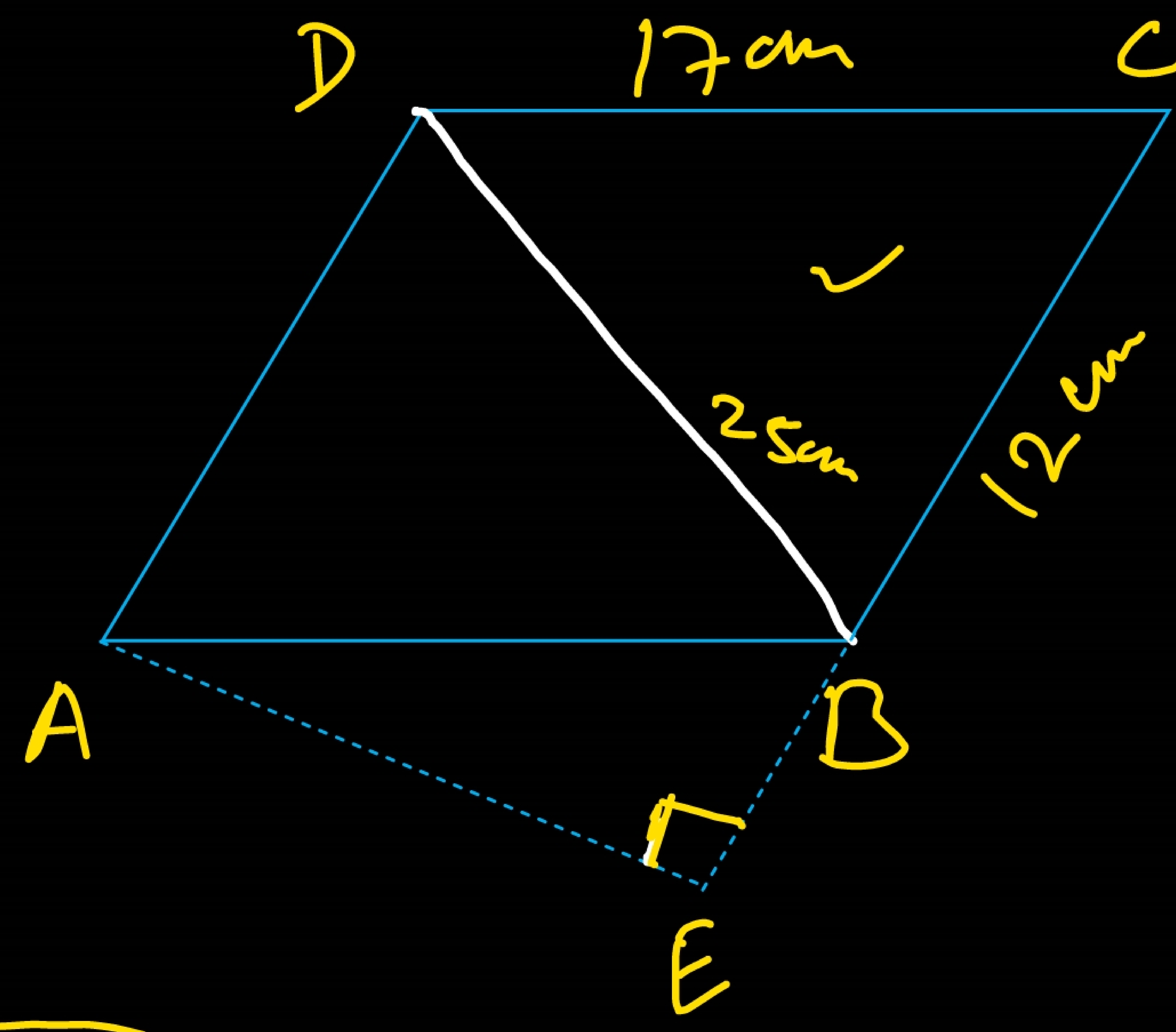
Ar. of Rhombus field $= \frac{1}{2}$ (Product of its diagonal)

$$= \frac{1}{\cancel{2}} \times \overset{80}{\cancel{160}} \times 120 \ m^2$$

$$= 9600 \ m^2$$

EXAMPLE 17

Find the area of the parallelogram ABCD in which BC = 12cm, CD = 17 cm and BD = 25 cm. Also, find the length of the altitude AE from vertex A on the side BC



S. Perimeter of $\Delta BCD = \dfrac{25+12+17}{2}$

$= \cancel{\dfrac{54}{2}}$

$= 27$ cm

$\therefore$ Ar of ||gm $= 2 \times$ Ar. of ΔBCD

$= 2 \times \sqrt{27(27-25)(27-12)(27-17)}$

$= 2 \times \sqrt{9 \times 3 \times 2 \times 3 \times 5 \times 2 \times 5}$

Ar. of ||gm ABCD $= 2 \times 3 \times 3 \times 2 \times 5$

$= 180 \text{ cm}^2$

$\therefore$ Ar. of ||gm ABCD $= 180 \text{ cm}^2$

$\Rightarrow$ Base $\times$ Height $= 180 \text{ cm}^2$

$\Rightarrow BC \times AE = 180 \text{ cm}^2$

$\Rightarrow 12 \times AE = 180$

$\Rightarrow AE = \dfrac{\cancel{180}^{\cancel{30}\ 15}}{\cancel{12}_{\cancel{2}}}$

$\therefore AE = 15 \text{ cm}$

EXAMPLE18

The adjacent sides of a parallelogram are 36 cm and 27 cm in length. If the distance between the shorter sides is 12 cm, find the distance between the longer sides.

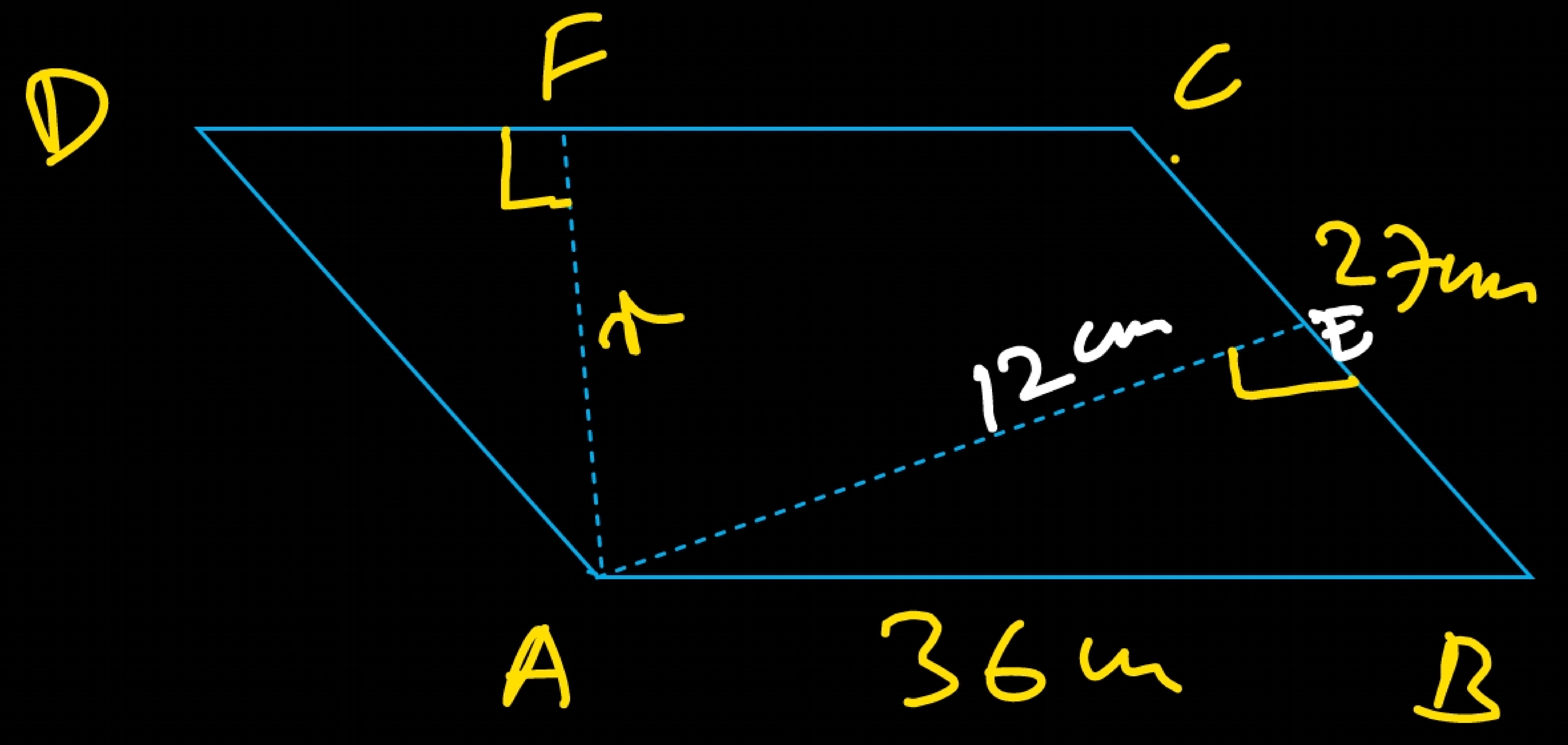
Ar. of || ABCD = Base x height

$\Rightarrow \quad CD \times x = BC \times 12$

$\Rightarrow \quad 36 \times x = 27 \times 12$

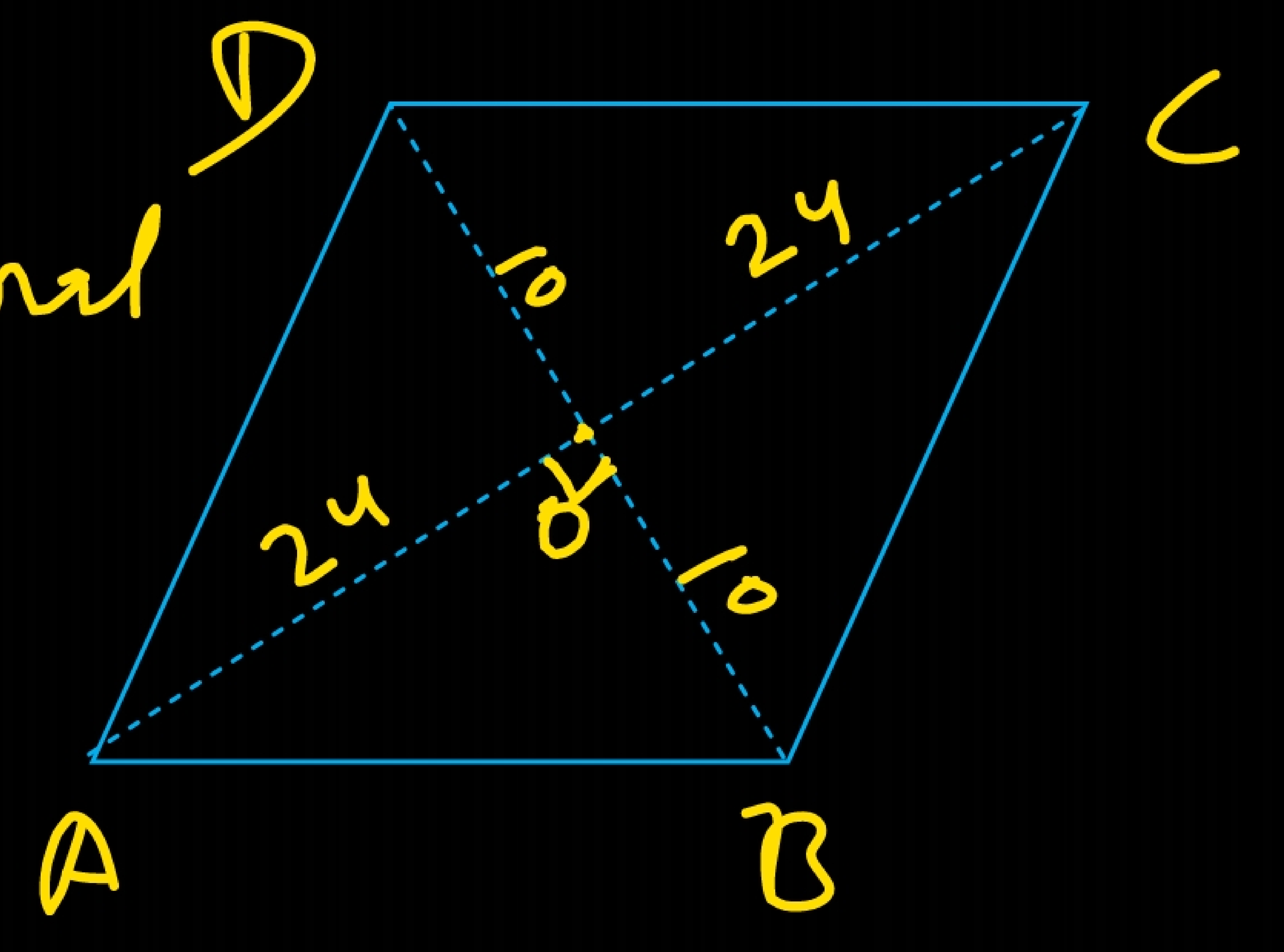
$\Rightarrow \quad x = \dfrac{\overset{9}{\cancel{27}} \times \cancel{12}}{\underset{\cancel{3}}{\cancel{36}}}$

$\Rightarrow \quad \boxed{x = 9\,\text{cm}}$



EXAMPLE 19

The diagonals of a rhombus are 48 cm and 20 cm long. Find (i) the area of the rhombus and (ii) the perimeter of the rhombus.



(i) Ar. of rhombus = $\frac{1}{2}$ Product of of its diagonal

$= \frac{1}{\cancel{2}} \times 48 \times \overset{10}{\cancel{20}}$

$= 480 \text{ cm}^2$

$\Rightarrow AB^2 = AO^2 + BO^2$

$\Rightarrow AB = \sqrt{24^2 + 10^2}$

$\Rightarrow AB = \sqrt{576 + 100}$

$\Rightarrow AB = \sqrt{676}$

$\Rightarrow AB = 26\ cm$

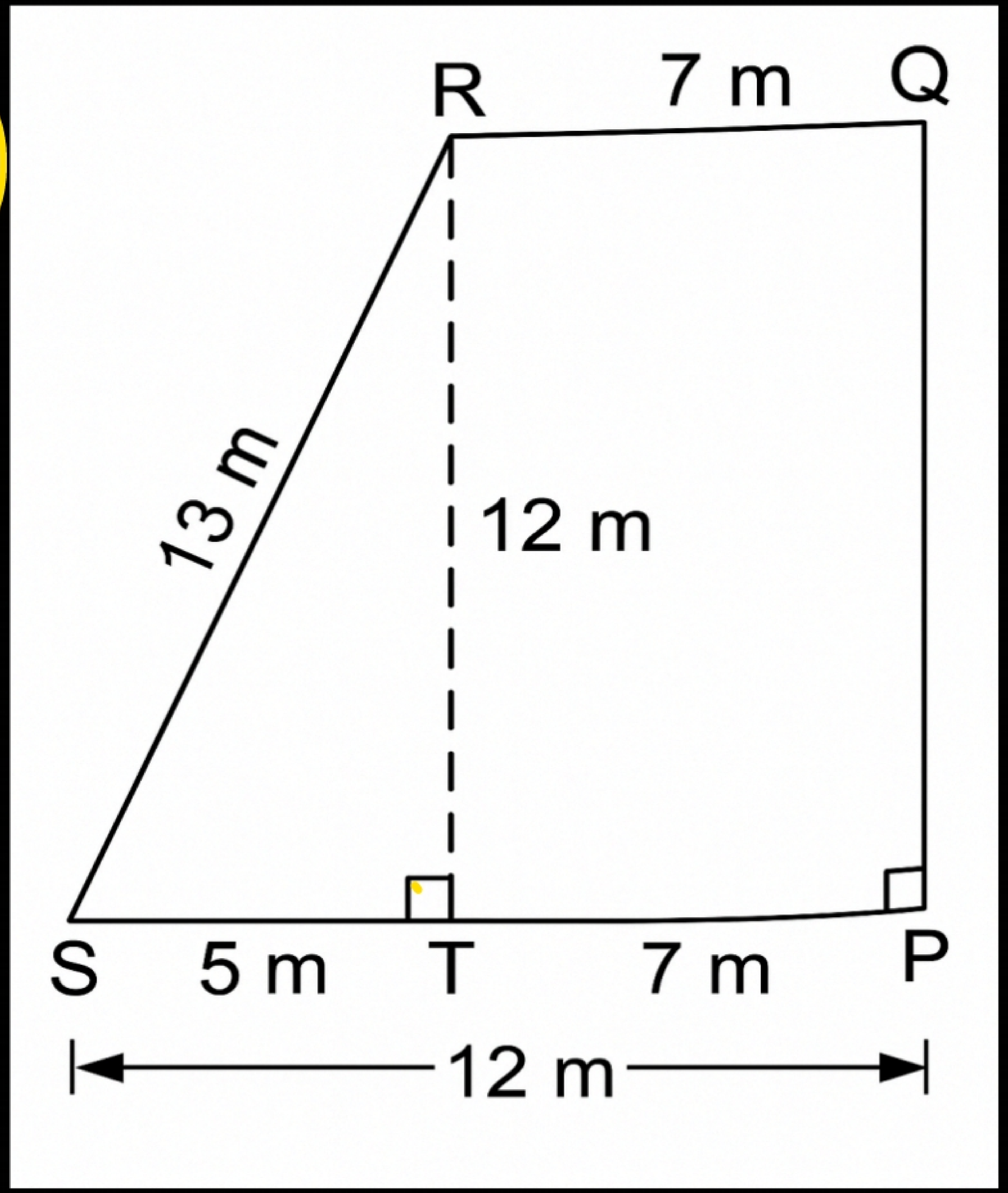
$\therefore$ Perimeter of rhombus $= 4 \times$ side

$= 4 \times 26\ cm$

$= 104\ cm$

EXAMPLE 20

Find the area of the given trapezium PQRS in which RQ || SP and PQ ⊥ SP such that RQ = 7 m, RS = 13 m and SP = 12 m.



$\therefore \; RT = \sqrt{13^2 - 5^2}$

$RT = \sqrt{169 - 25}$

$RT = \sqrt{144}$

$\boxed{RT = 12\ m}$

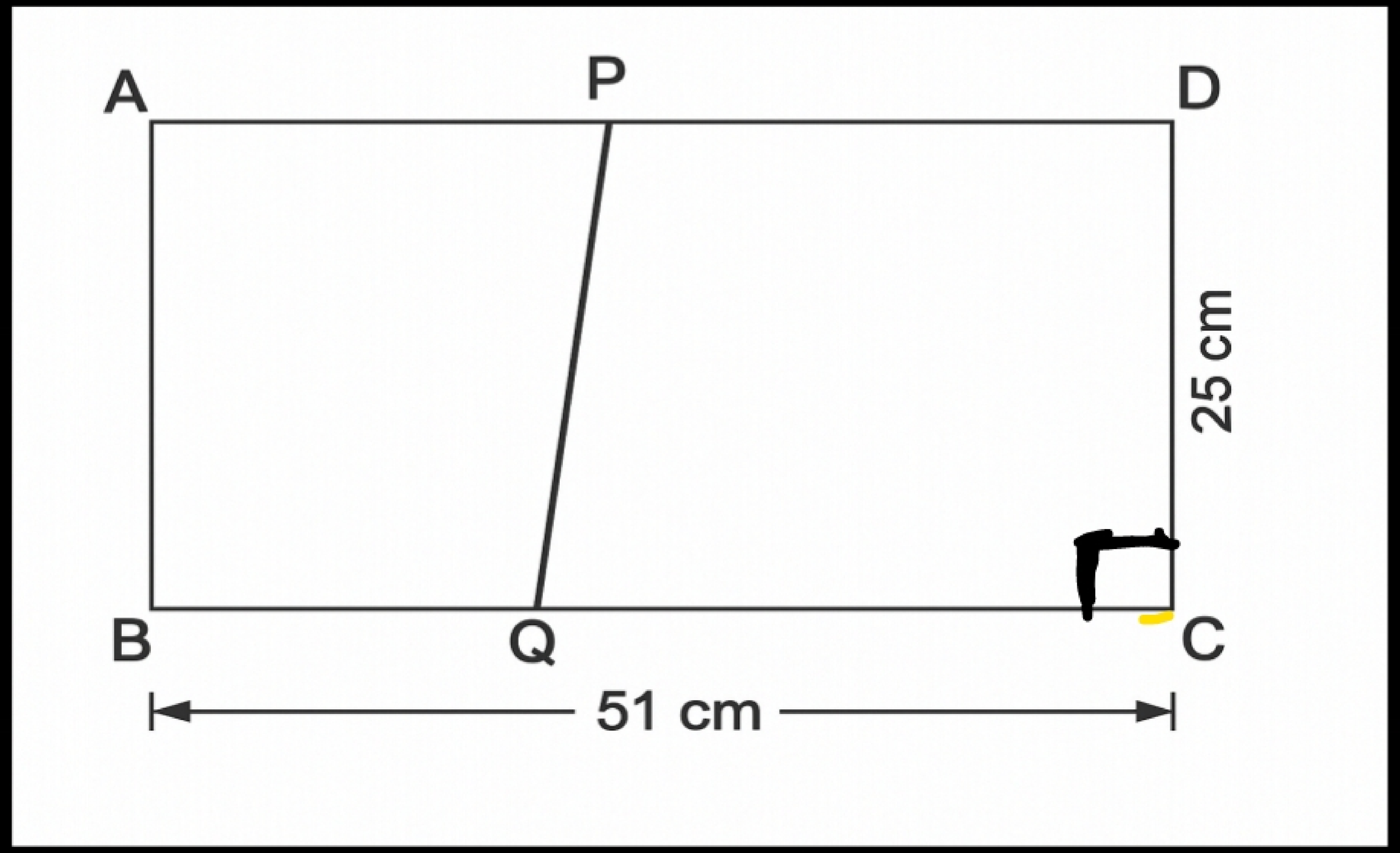
Ar. of trapezium $= \frac{1}{2}$ (sum of || sides) $\times h$

$= \frac{1}{\cancel{2}} \times (12 + 7) \times \cancel{12}^{6}$

$= 114\ m^2$

EXAMPLE 21

In the given figure, ABCD is a rectangle of length 51 cm and breadth 25 cm. A trapezium PQCD with its parallel sides QC and PD in the ratio 9 : 8 is cut off from the rectangle, as shown in the figure. If the area of the trapezium PQCD is ⅚th part of the area of the rectangle, find the lengths QC and PD.



Let the ratio be x

$\therefore QC = 9x$ and $PD = 8x$

Ar. of trapezium $= \frac{5}{6}$ of Ar. of rectangle

$\Rightarrow \frac{1}{2}(9x + 8x) \times 25 = \frac{5}{6} \times 51 \times 25$

$\Rightarrow 17x = \frac{5}{\cancel{6}_{3}} \times \frac{\cancel{51}^{17} \times \cancel{25} \times \cancel{2}}{\cancel{25}}$

$$\therefore\ x = \frac{\cancel{17} \times 5}{\cancel{17}}$$

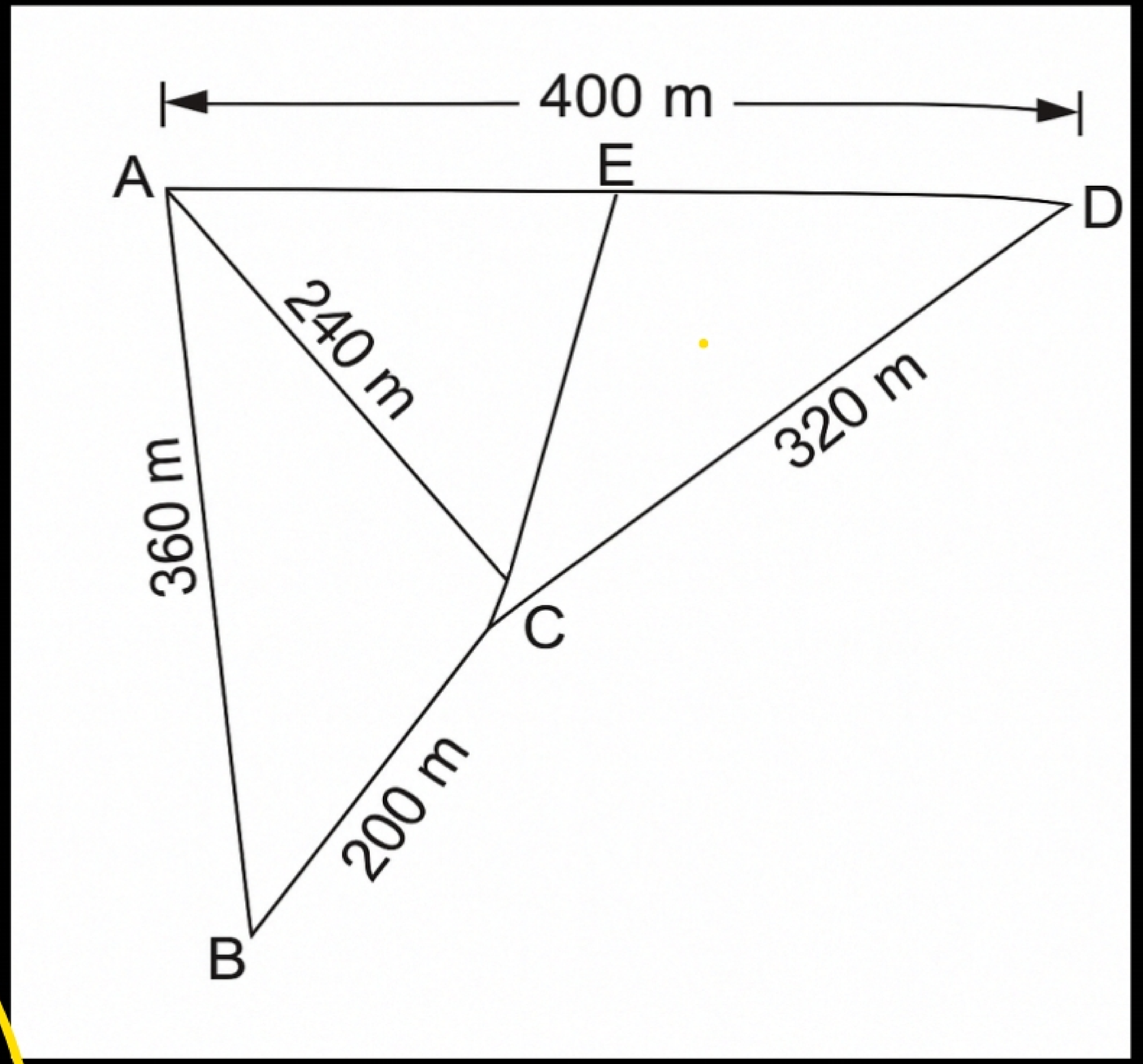
$$\boxed{x = 5}$$

$$\therefore\ QC = 9x$$
$$= 9 \times 5$$
$$= 45\ cm$$

$$\therefore\ PD = 8x$$
$$= 8 \times 5$$
$$= 40\ cm$$

EXAMPLE 22

A farmer has a triangular field with sides 360 m. 200 m and 240 m, where he grows wheat. Adjacent to this field, he has another triangular field with sides 240 m, 320 m and 400 m, divided into two parts by joining the midpoint of the longest side to the opposite vertex. He grows potatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 10000 m².)



Semi perimeter of $\triangle ABC = \dfrac{360+200+240}{2}$

$S = 400$

Ar. of $\triangle ABC = \sqrt{400(400-360)(400-200)(400-240)}$

$= \sqrt{400 \times 40 \times 200 \times 160}$

Ar. of $\triangle ABC = \sqrt{400 \times 4 \times \underline{10} \times 100 \times 2 \times 16 \times \underline{10}}$

$$= 20 \times 2 \times 10 \times 10 \times 4 \sqrt{2}$$

$$= 16000 \sqrt{2} \ m^2$$

$$= \frac{16\cancel{000} \times 1.41}{10\cancel{000}} \text{ hec.}$$

$$= \frac{22.56}{10}$$

$$= 2.256 \text{ hec.} \qquad = 2.26 \text{ he.}$$

In ΔACD

Semiperimeter of $\Delta ACD = \dfrac{320 + 240 + 400}{2}$

$= 480 \text{ m}$

Ar. of $\Delta ACD = \sqrt{480(480-320)(480-240)(480-400)}$

$= \sqrt{16 \times 30 \times 16 \times 10 \times 24 \times 10 \times 80}$

$= \sqrt{16 \times 2 \times 3 \times 5 \times 16 \times 10 \times 10 \times 4 \times 2 \times 3 \times 10 \times 4 \times 2}$

$= 16 \times 2 \times 3 \times 10 \times 4 \times 10$

$$= 128 \times 3 \times 100$$

$$= 38400 \ m^2$$

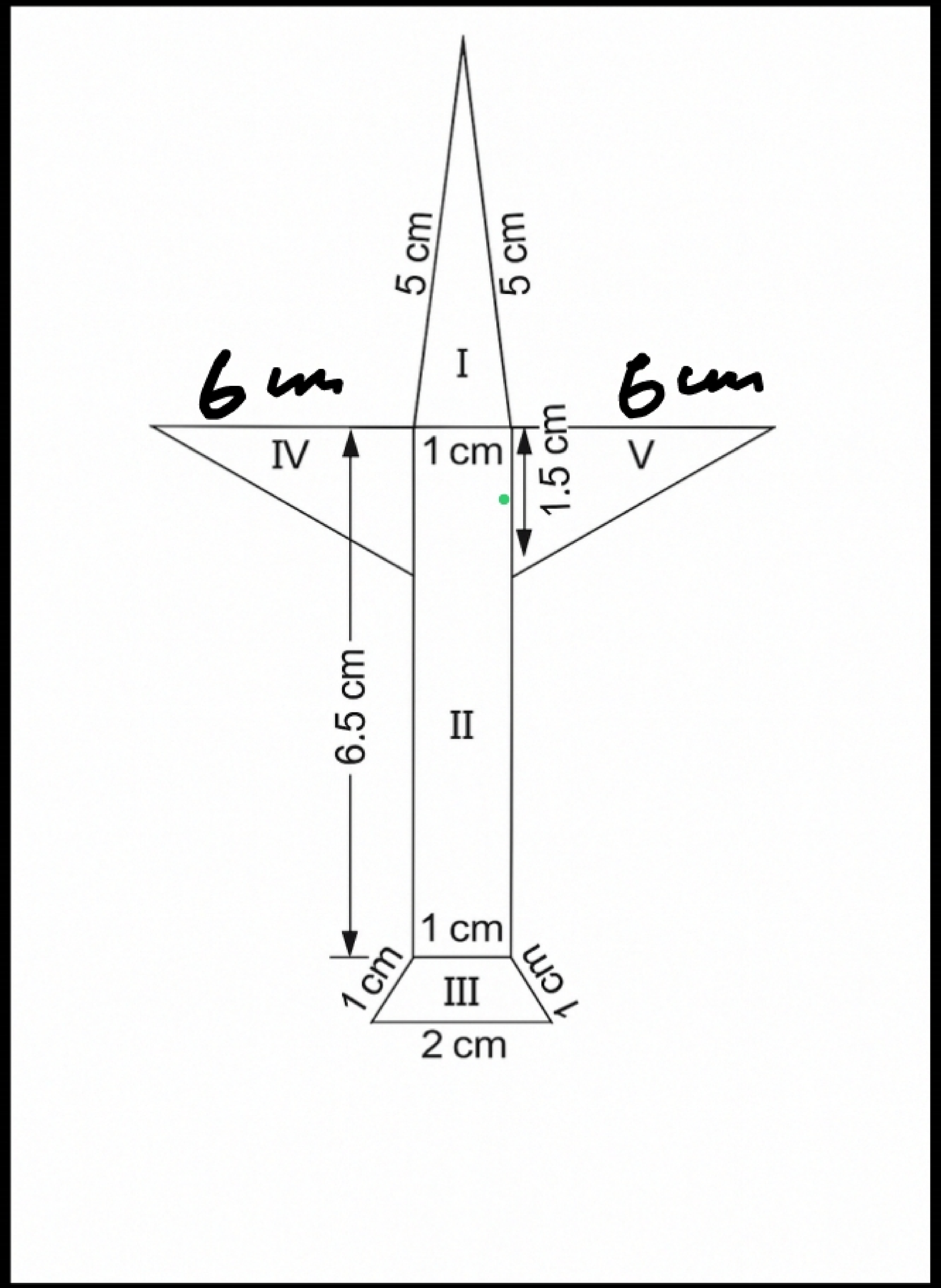
In hec. $= \dfrac{384\cancel{00}}{100\cancel{00}}$

Ar. of Δ ACD $= 3.84$ hec.

Now, Ar of Δ ACE $=$ Ar. of Δ CDE $= 1.92$ hc.

EXAMPLE 23

Reenu made a picture of an aeroplane with coloured paper as shown in the figure given below. Find the total area of the paper used.



Ar. of part I $= \frac{b}{4}\sqrt{4a^2 - b^2}$

$= \frac{1}{4}\sqrt{4 \times 5^2 - 1^2}$

$= \frac{1}{4}\sqrt{99}$

$= \frac{1}{4}\sqrt{9 \times 11}$

$= \frac{3}{4} \times \sqrt{11}$

$= \frac{3}{4} \times 3.316$

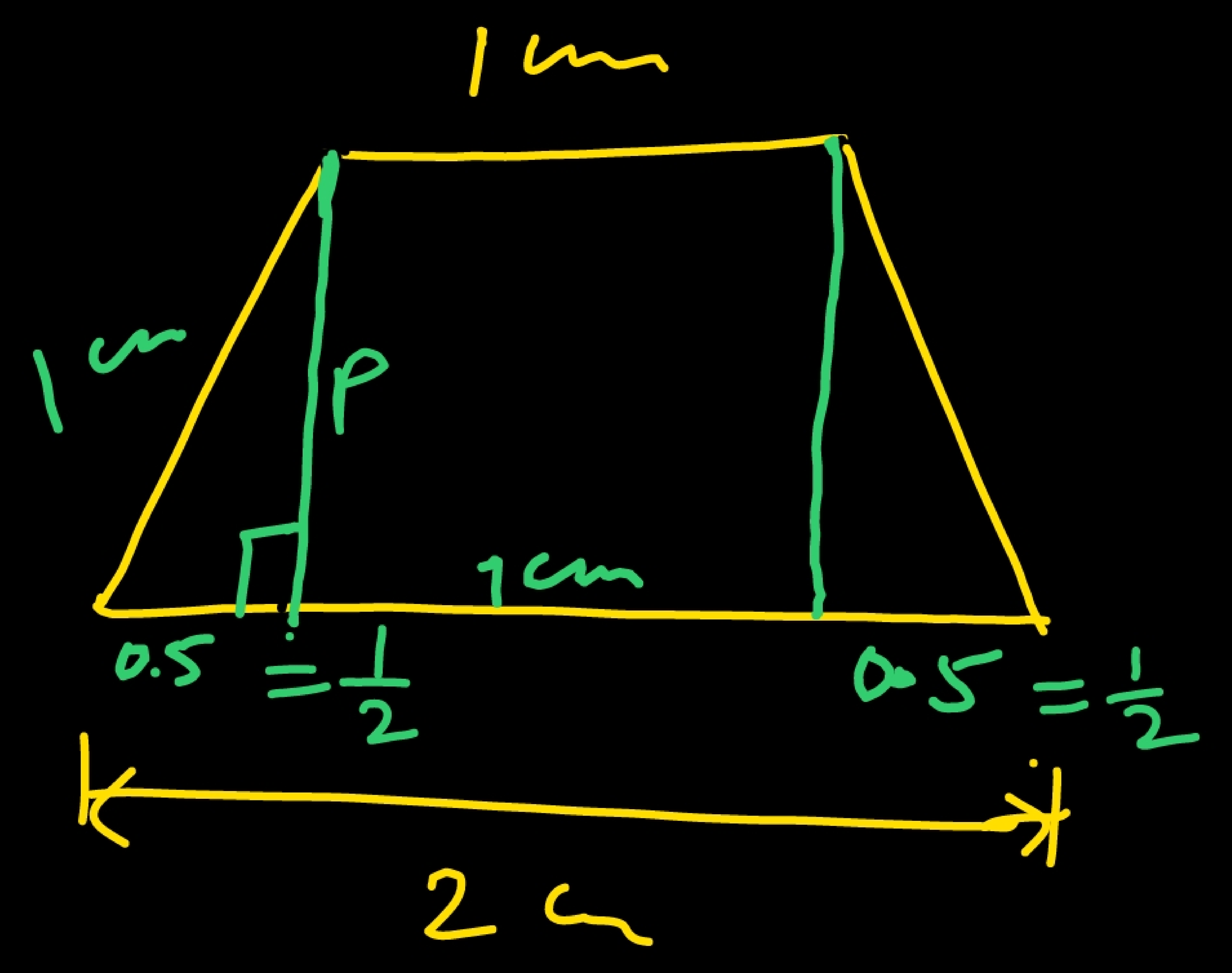
$= \frac{9.948}{4}$

$= 2.487 \text{ cm}^2$

Ar. of II $= L \times B$

$= 6.5 \times 1$

$= 6.5\ cm^2$



Ar. of IV $= \frac{1}{2}$ (sum of ∥ side) h

$= \frac{1}{2}(2+1) \times \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{4}$

$= \frac{3 \times 1.732}{4} = 1.299\ cm^2$

$= 1.3\ cm^2$

$p = \sqrt{1^2 - \left(\frac{1}{2}\right)^2}$

$p = \sqrt{1 - \frac{1}{4}}$

$p = \sqrt{\frac{3}{4}}$

$p = \frac{\sqrt{3}}{2}$

$$\text{Ar. of } \overline{\underline{IV}} + \underline{V} = 2 \times \frac{1}{\cancel{2}} \times 1.5 \times 6$$

$$= 9 \text{ cm}^2$$

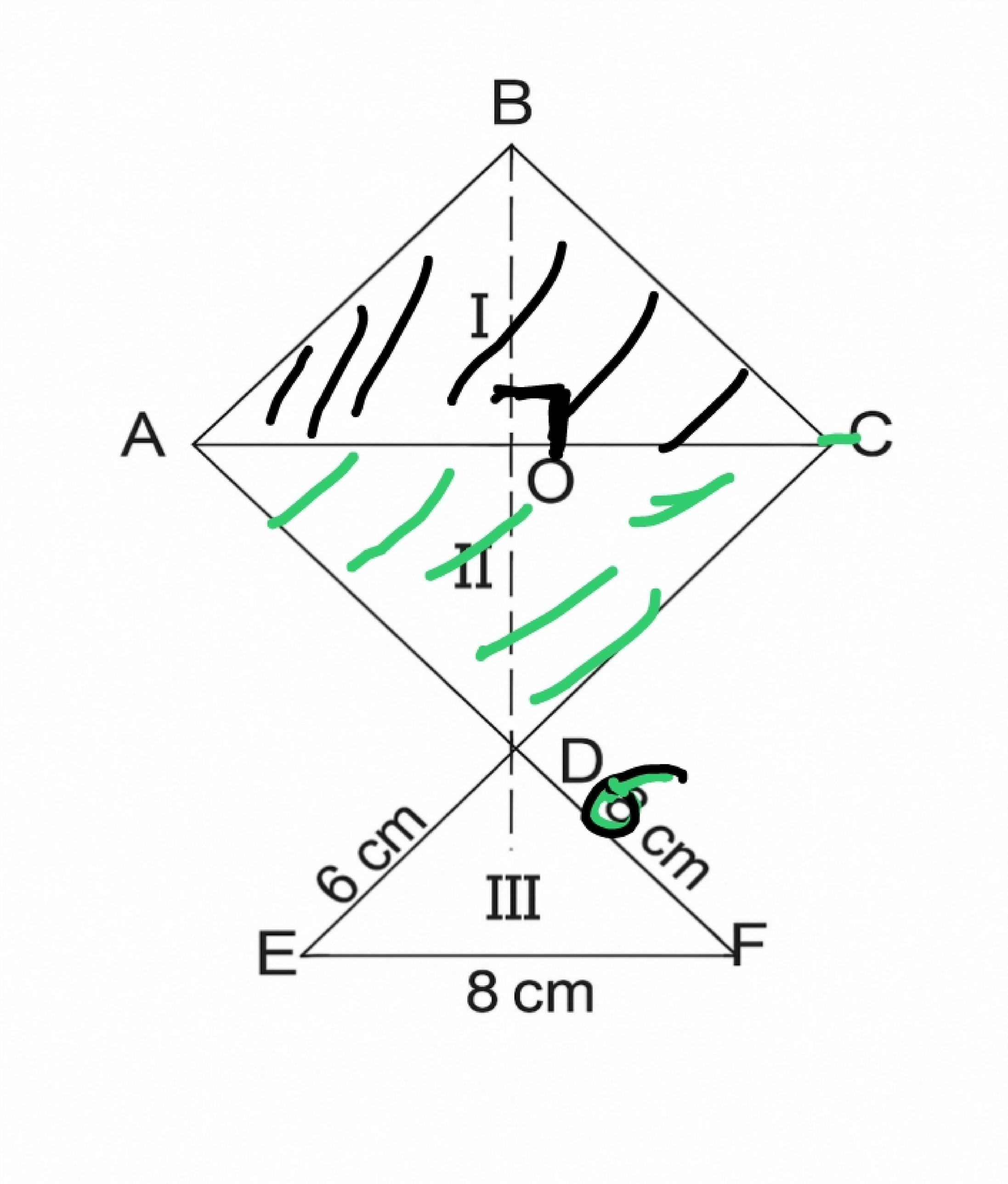
$$\text{Ar. of Aeroplane} = \text{Ar}\left(I + \overline{\underline{II}} + \overline{\underline{III}} + \overline{\underline{IV}} + V\right)$$

$$= \left(2.487 + 6.5 + 1.3 + 9\right)$$

$$= 19.287 \text{ cm}^2$$

EXAMPLE 24

A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in the figure. How much paper of each shade has been used in it?



$$\text{Ar. of I} = \text{Ar. of II} = \frac{1}{2} \times AC \times OB$$

$$= \frac{1}{\cancel{2}} \times 32 \times \cancel{16}^{8}$$

$$= 256\ cm^2$$

$$\text{Ar. of III} = \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{8}{4}\sqrt{4 \times 6^2 - 8^2}$$

Ar. of (III) $= 2\sqrt{4 \times 36 - 64}$

$= 2\sqrt{144 - 64}$

$= 2 \times \sqrt{80}$

$= 2 \times \sqrt{16 \times 5}$

$= 2 \times 4 \times \sqrt{5}$

$= 8 \times 2.236$

$= 17.968 \text{ cm}^2$

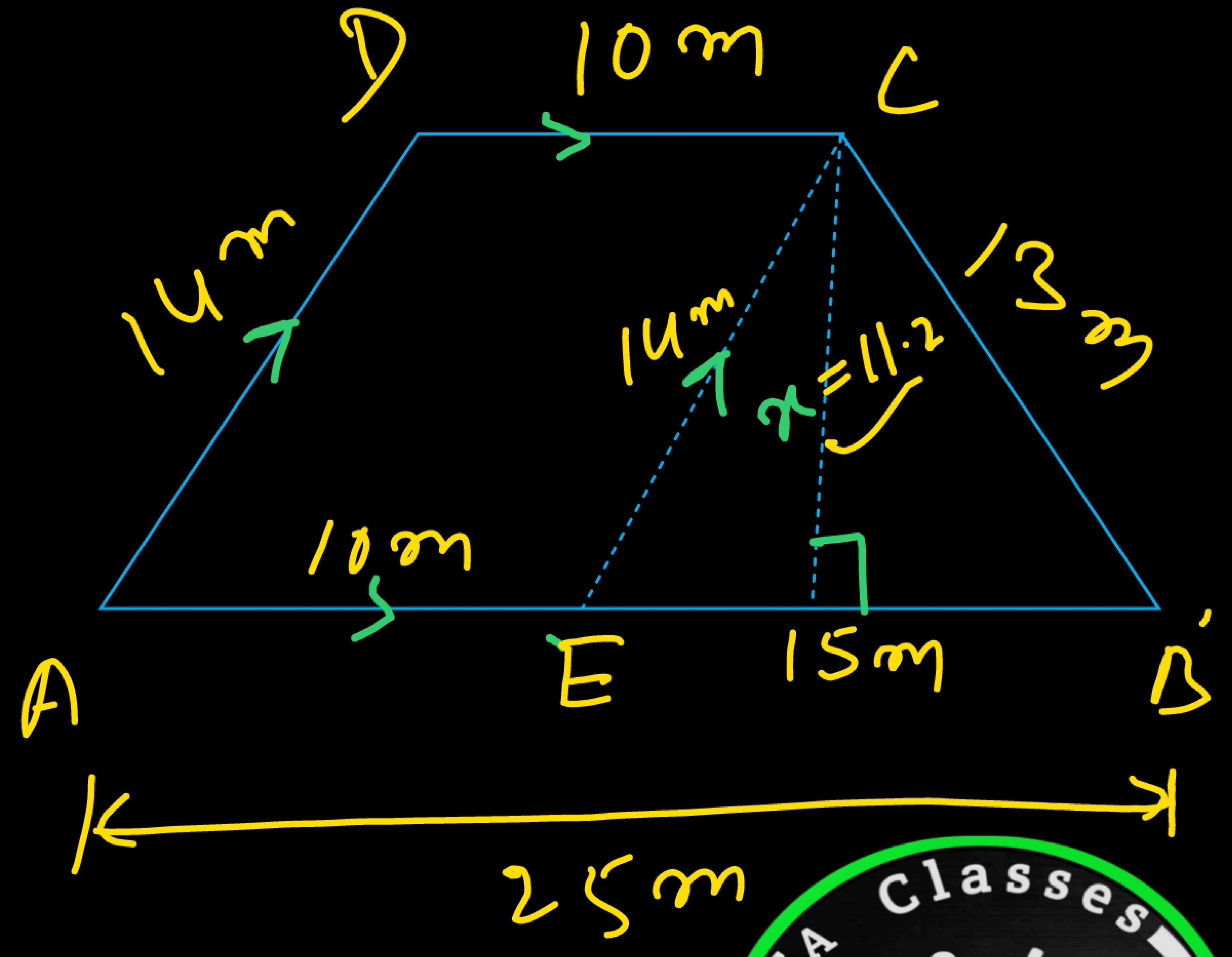
EXAMPLE 25

A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m and the nonparallel sides are 14 m and 13 m. Find the area of the field.

S. Perimeter of $\Delta BCE = \dfrac{15+13+14}{2}$

$= \cancel{\dfrac{42}{2}}$

$= 21\,m$



Ar. of $\Delta BCE = \sqrt{21(21-15)(21-14)(21-13)}$

$= \sqrt{21 \times 6 \times 7 \times 8}$

$$= \sqrt{3 \times 7 \times 2 \times 3 \times 7 \times 4 \times 2}$$

$$= 3 \times 7 \times 2 \times 2$$

Ar. of $\Delta BCE = 84\ m^2$

$$\Rightarrow \frac{1}{2} \times BE \times x = 84$$

$$\Rightarrow \frac{1}{2} \times 15 \times x = 84$$

$$\Rightarrow x = \frac{84 \times 2}{15}$$

$$\Rightarrow x = \frac{168}{15}$$

$$\Rightarrow \boxed{x = 11.2\ m}$$

Ar. of trap. $= \frac{1}{2}$ (Sum of $\parallel$ side) h

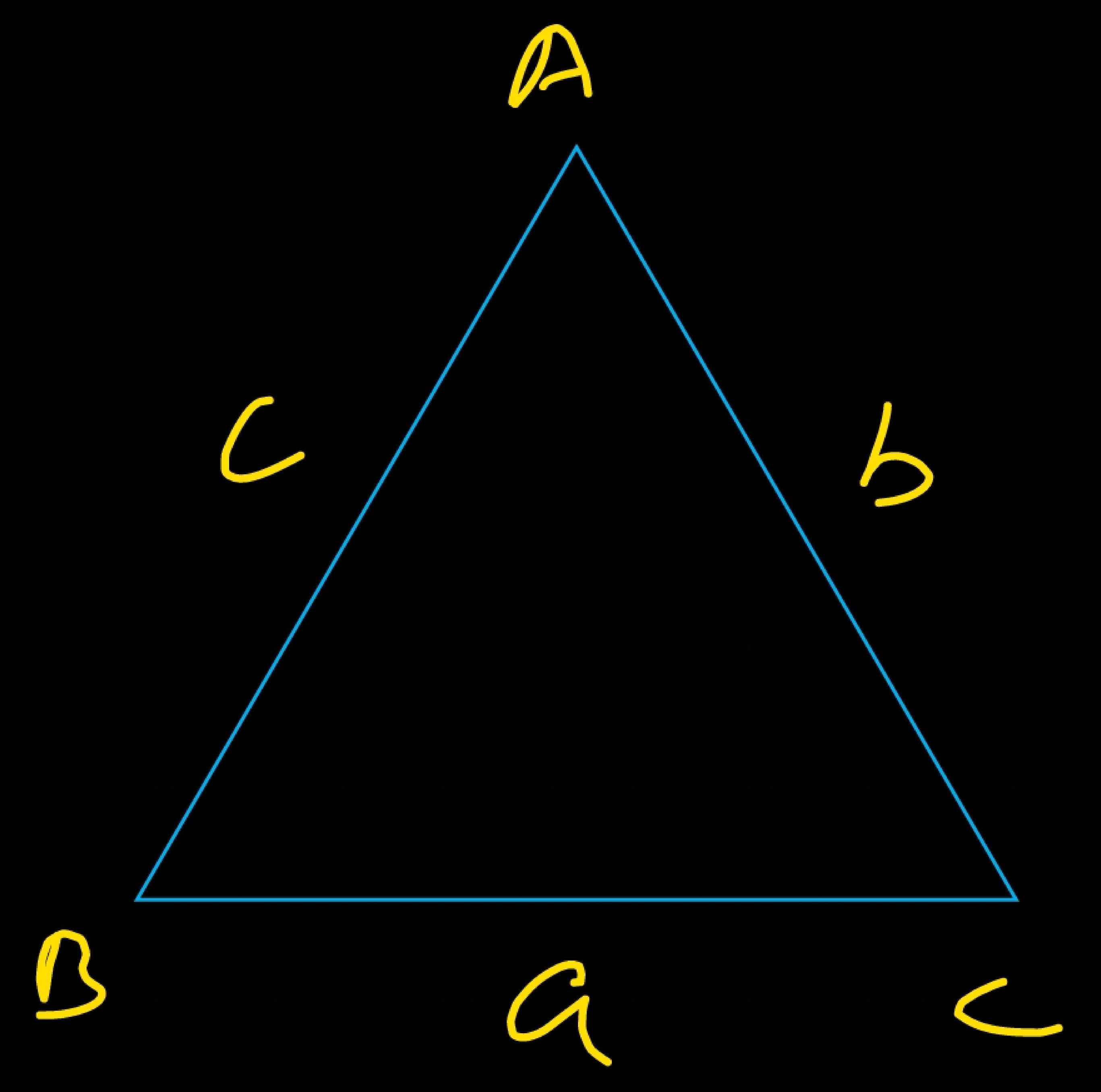
$= \frac{1}{2} \times (25 + 10) \times 11.2$

$= \frac{1}{\cancel{2}} \times \overset{17.5}{\cancel{35}} \times 11.2$

$= 196\ m^2$

EXAMPLE 26

If each side of a triangle is doubled then find the ratio of the area of the new triangle thus formed and the given triangle.



Let the side of the triangle be a, b and c

and Semiperimeter be S

$\therefore$ Ar. of triangle $= \sqrt{S(S-a)(S-b)(S-c)}$

Now

Side of triangle are doubled

$\therefore$ New sides $= 2a$, $2b$ and $2c$

$\therefore$ New Semiperimeter $= 2S$

$$\text{Ar. of New triangle} = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$= \sqrt{2s \times 2 \times 2 \times 2(s-a)(s-b)(s-c)}$$

$$= \sqrt{16\, s(s-a)(s-b)(s-c)}$$

$$= 4\sqrt{s(s-a)(s-b)(s-c)}$$

$\therefore$ Ar. of New triangle $= 4 \times$ Ar. of given triangle.

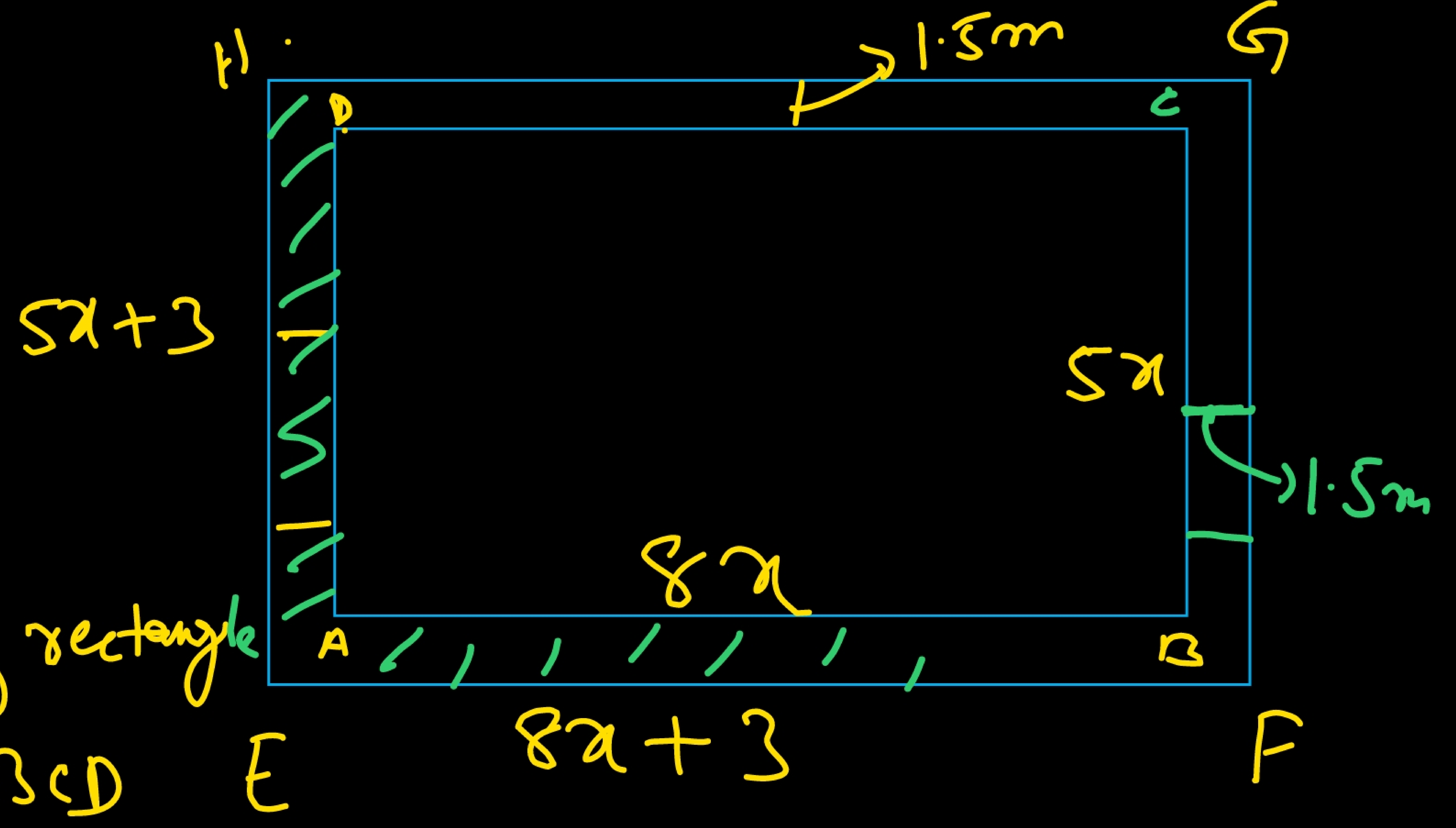
EXAMPLE 27

The length and breadth of a rectangular park are in the ratio 8:5 A path, 1.5 m wide, running all around the outside of the park has an area of $594 m^2$. Find the dimensions of the park.

Let the ratio be x

$\therefore$ lenght of rectangle $= 8x$

breadth $= 5x$



Ar. of Path = Ar. of rectangle EFGH − Ar. of rectangle ABCD

$\Rightarrow 594 = (8x+3)(5x+3) - (8x \times 5x)$

$\Rightarrow 594 = \cancel{40x^2} + 24x + 15x + 9 - \cancel{40x^2}$

$$\Rightarrow 594 - 9 = 39x$$

$$\Rightarrow \frac{\cancel{585}^{\cancel{45}\,15}}{\cancel{39}_{3}} = x$$

$$\Rightarrow \boxed{x = 15\text{ m}}$$

$\therefore$ Length of rectangle $= 8x$

$$= 8 \times 15 \text{ m}$$

$$= 120\text{ m}$$

$\therefore$ Breadth of rectangle $= 5x$

$$= 5 \times 15 \text{ m}$$

$$= 75\text{ m}$$