RS AGGARWAL

Exercise 13 A

Examples 1 to 41

One Shot



Examples 01:-

Prove that

(i) $(1-\sin^2\theta)\sec^2\theta = 1$

(ii) $(\operatorname{cosec}^2\theta - 1)\tan^2\theta = 1$

(iii) $(\sec^2\theta - 1)(1-\operatorname{cosec}^2\theta) = -1$

(iv) $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta) = 1$

Solution:-

(i) $(1-\sin^2\theta)\cdot\sec^2\theta = 1$

L.H.S $= (1-\sin^2\theta)\cdot\sec^2\theta$

$= \cos^2\theta \times \sec^2\theta$ $\quad \{\sin^2\theta+\cos^2\theta = 1\}$, $\{\cos^2\theta = 1-\sin^2\theta\}$

$= 1 =$ R.H.S Proved $\{\because \cos\theta \times \sec\theta = 1\}$

Examples 01:-

$$(ii)\ (\operatorname{cosec}^2\theta - 1)\tan^2\theta = 1$$



L.H.S $= (\operatorname{cosec}^2\theta - 1)\cdot \tan^2\theta$

$= \cot^2\theta \times \tan^2\theta$ $\quad \{\operatorname{cosec}^2\theta - \cot^2\theta = 1\}$

$= 1 = \text{R.H.S}$

Proved.. $\quad \{\because \tan\theta \cdot \cot\theta = 1\}$

Examples 01:-

$$(iii)\ (\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta) = -1$$

$\underline{L.H.S} = (\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta)$

$= \tan^2\theta \{-(\operatorname{cosec}^2\theta - 1)\}$ $\{\because \sec^2\theta - \tan^2\theta = 1$ & $\sec^2\theta - 1 = \tan^2\theta\}$

$= \tan^2\theta \times [-\cot^2\theta]$ $\{\because \operatorname{cosec}^2\theta - \cot^2\theta = 1\}$

$= -\tan^2\theta \times \cot^2\theta$

$= -1$ $\{\because \tan\theta \cdot \cot\theta = 1\}$

Examples 01:-

$$(iv)\ \sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)=1$$



L.H.S :- $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)$

$= \frac{1}{\cos\theta}\left(1-\sin\theta\right)\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)$

$\left\{\because \sec\theta=\frac{1}{\cos\theta}\right.$
$\left.\&\ \tan\theta=\frac{\sin\theta}{\cos\theta}\right\}$

$= \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1-\sin^2\theta}{\cos^2\theta}$

$\left\{\because \sin^2\theta+\cos^2\theta=1\right.$
$\left.\cos^2\theta=1-\sin^2\theta\right\}$

$= \frac{\cos^2\theta}{\cos^2\theta} = 1$ = R.H.S Proved

Examples 02:-

Prove that

(i) $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$

(ii) $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

(iii) $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta \operatorname{cosec}^2\theta$

① $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \cdot \operatorname{cosec}^2\theta$

L.H.S $= \sec^2\theta + \operatorname{cosec}^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \cdot \sin^2\theta}$

$\left\{ \because \sec\theta = \dfrac{1}{\cos\theta} \right.$
$\left. \& \operatorname{cosec}\theta = \dfrac{1}{\sin\theta} \right\}$

$$= \frac{1}{\cos^2\theta \times \sin^2\theta} \qquad \{\sin^2\theta + \cos^2\theta = 1\}$$

$$= \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta \times \operatorname{cosec}^2\theta = R.H.S$$

Proved.

Examples 02:-

(ii) $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$



L.H.S $\tan^2\theta - \sin^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$ $\quad \{\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\}$

$= \dfrac{\sin^2\theta - \sin^2\theta \cdot \cos^2\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta (1 - \cos^2\theta)}{\cos^2\theta}$

$\because \dfrac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta$ $\quad \{1 - \cos^2\theta = \sin^2\theta\}$

$= \tan^2\theta \times \sin^2\theta = R.H.S$

Proved !!

Examples 02:-

(iii) $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta \operatorname{cosec}^2\theta$



L.H.S $\tan^2\theta + \cot^2\theta + 2$

$= \sec^2\theta - 1 + \operatorname{cosec}^2\theta - 1 + 2$

$= \sec^2\theta + \operatorname{cosec}^2\theta - 2 + 2$

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \cdot \sin^2\theta}$

$\{\because \sec^2\theta - 1 = \tan^2\theta$ & $\operatorname{cosec}^2\theta - 1 = \cot^2\theta\}$

$\{\sec\theta = \frac{1}{\cos\theta}$ & $\operatorname{cosec}\theta = \frac{1}{\sin\theta}\}$

$$= \frac{1}{\cos^2\theta \cdot \sin^2\theta}$$

$$= \sec^2\theta \cdot \operatorname{cosec}^2\theta = R.H.S \quad \text{Proved}$$

Examples 03:-

Prove that

$$(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)=2.$$

[CBSE 2007, '08, '17, '19]

$$\text{L.H.S} = (1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)$$

$$= \left[1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right]\left[1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right]$$

$$= \left[\frac{\sin A+\cos A-1}{\sin A}\right]\left[\frac{\cos A+\sin A+1}{\cos A}\right]$$

$$= \frac{\sin A\cdot\cos A+\sin^2 A+\sin A+\cos^2 A+\sin A\cdot\cos A+\cos A-\cos A-\sin A-1}{\sin A\cdot\cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cdot \cos A - 1}{\sin A \cdot \cos A}$$

$$= \frac{1 + 2 \sin A \cdot \cos A - 1}{\sin A \cdot \cos A}$$

$$= \frac{2 \sin A \cdot \cos A}{\sin A \cdot \cos A}$$

$$= 2 = \text{R.HS Prove}$$

Examples 04:-

Prove that

$$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = (7 + \tan^2\theta + \cot^2\theta).$$

[CBSE 2015, '16, '19, '20]

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LHS :- $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta \cdot \operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta \cdot \sec\theta$

$= \sin^2\theta + \cos^2\theta + 1 + \cot^2\theta + 2 \times 1 + 1 + \tan^2\theta + 2 \times 1$

$= 1 + 1 + 2 + 1 + 2 + \cot^2\theta + \tan^2\theta$

$= 7 + \tan^2\theta + \cot^2\theta$ = RHS Proved !!

$\{\because \sin\theta \operatorname{cosec}\theta = 1$
$\cos\theta \times \sec\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \operatorname{cosec}^2\theta\}$

Examples 05:-

Prove that

$$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = \frac{1}{(\tan\theta + \cot\theta)}$$ [CBSE 2009C, '10]

LHS :- $(\operatorname{Cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)$

$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)$

$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)$

$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta}$

$= \cos\theta \cdot \sin\theta$

R.HS $= \frac{1}{\tan\theta + \cot\theta}$

$= \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta}}$

$$= \frac{1}{\frac{1}{\sin\theta \cdot \cos\theta}}$$

$$= 1 \times \frac{\sin\theta \cdot \cos\theta}{1}$$

$$= \sin\theta \cdot \cos\theta = \text{R.H.S (Proved)} \therefore$$

Examples 06:-

Prove that

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta.$$

$$\text{L.H.S} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\left(1 - 2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$

$$= \frac{\sin\theta}{\cos\theta}\,\frac{\left[1 - 2\left(1 - \cos^2\theta\right)\right]}{\left(2\cos^2\theta - 1\right)}$$

$$= \tan\theta\,\frac{\left(1 - 2 + 2\cos^2\theta\right)}{2\cos^2\theta - 1}$$

$$= \tan\theta\,\frac{\left(2\cos^2\theta - 1\right)}{\left(2\cos^2\theta - 1\right)}$$

$$= \tan\theta = \text{R.H.S}$$ Proved.

Examples 07:-

Prove that $\dfrac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta=1.$



$$\text{L.H.S} = \frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cdot\cos\theta \qquad \{a^3+b^3=[(a+b)(a^2+b^2-ab)]\}$$

$$= \frac{(\sin\theta+\cos\theta)[\sin^2\theta+\cos^2\theta-\sin\theta\cdot\cos\theta]}{(\sin\theta+\cos\theta)}+\sin\theta\cdot\cos\theta$$

$$= 1-\sin\theta\cdot\cos\theta+\sin\theta\cdot\cos\theta$$

$$= 1 = \text{R.H.S Proved.}$$

Examples 08

Prove that $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$.



L.H.S $= (\operatorname{cosec}\theta - \cot\theta)^2$

$= \left[\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right]^2$

$= \left[\dfrac{1-\cos\theta}{\sin\theta}\right]^2$

$= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos^2\theta)}$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1-\cos\theta}{1+\cos\theta}$ = RHS Proved

Examples 09

Prove that

$$\frac{\cos\theta}{(1-\tan\theta)}+\frac{\sin\theta}{(1-\cot\theta)}=(\cos\theta+\sin\theta).$$

[CBSE 2009, '13]

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$$\text{L.H.S} = \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}} + \frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}$$

$$= \cos\theta \times \frac{\cos\theta}{\cos\theta-\sin\theta} + \sin\theta \times \frac{\sin\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{-(\cos\theta-\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{(\cos\theta - \sin\theta)}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)}$$

$$= \cos\theta + \sin\theta = \text{R.H.S Proved.}$$

Examples 10

Prove that

$$\frac{\tan A}{(1-\cot A)}+\frac{\cot A}{(1-\tan A)}=(1+\tan A+\cot A).$$ [CBSE 2010, '13]

L.H.S $= \dfrac{\tan A}{1-\dfrac{\cos A}{\sin A}} + \dfrac{\cot A}{1-\dfrac{\sin A}{\cos A}}$

$= \dfrac{\tan A}{\dfrac{\sin A-\cos A}{\sin A}} + \dfrac{\cot A}{\dfrac{\cos A-\sin A}{\cos A}}$

$= \dfrac{\sin A}{\cos A} \times \dfrac{\sin A}{(\sin A-\cos A)} + \dfrac{\cos A}{\sin A} \times \dfrac{\cos A}{\cos A-\sin A}$

$= \dfrac{\sin^2 A}{\cos A(\sin A-\cos A)} + \dfrac{\cos^2 A}{\sin A(\cos A-\sin A)}$

$= \dfrac{\sin^2 A}{\cos A(\sin A-\cos A)} + \dfrac{\cos^2 A}{-[\sin A(\sin A-\cos A)]}$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} - \frac{\cos^2 A}{\sin A(\sin A - \cos A)}$$

$$= \frac{1}{(\sin A - \cos A)}\left[\frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}\right]$$

$$= \frac{1}{(\sin A - \cos A)}\left[\frac{\sin^3 A - \cos^3 A}{\cos A \cdot \sin A}\right]$$

$$= \frac{1}{(\sin A - \cos A)} \left[\frac{(\sin A - \cos A) \times \{\sin^2 A + \cos^2 A + \sin A \cdot \cos A\}}{\sin A \cdot \cos A} \right.$$

$$= \frac{1 + \sin A \cdot \cos A}{\sin A \cdot \cos A}$$

$$= \frac{1}{\sin A \cdot \cos A} + \frac{\sin A \cdot \cos A}{\sin A \cdot \cos A}$$

$$= \frac{1}{\sin A \cdot \cos A} + 1$$

$$\underline{R.HS} = 1 + \tan A + \cot A$$

$$= 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}$$

$$= 1 + \frac{1}{\sin A \cdot \cos A} = L.HS$$

Proved !!

Examples 11

Prove that

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A.$$

$$= \frac{1+\tan^2 A}{1+\cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{Cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \boxed{\tan^2 A}$$

Now $\left(\frac{1-\tan A}{1-\cot A}\right)^2$

$$= \left[\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right]^2$$

$$= \left[\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}} \right]^2$$

$$= \left[\frac{\cos A - \sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} \right]^2$$

$$= \left[- \frac{(\sin A - \cos A)}{\cos A} \times \frac{\sin A}{(\sin A - \cos A)} \right]^2$$

$$= \left[- \frac{\sin A}{\cos A} \right]^2$$

$$= \left(- \tan A \right)^2$$

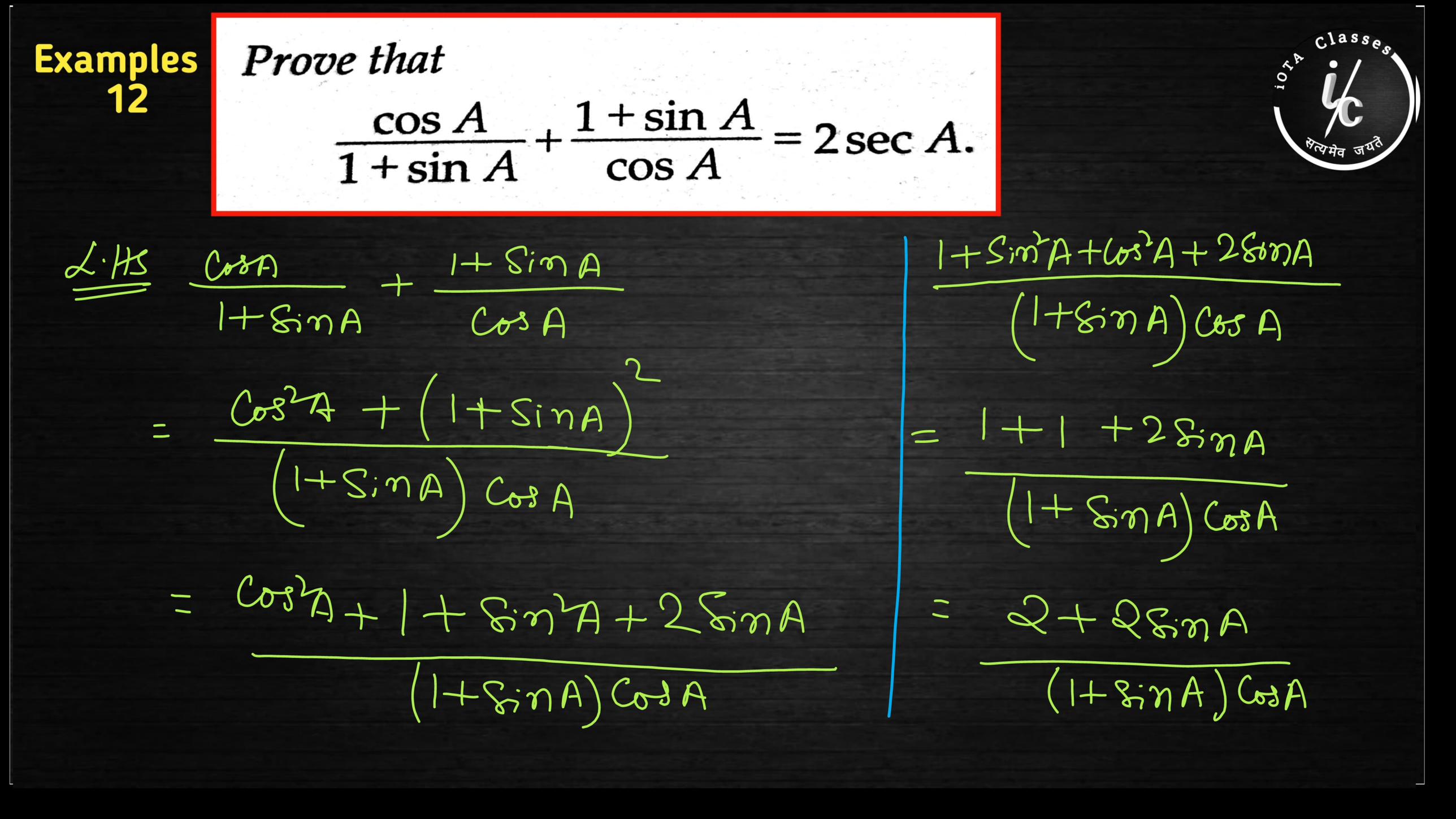
$$= \tan^2 A$$

Hence $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A} \right)^2 = \tan^2 A$

Examples 12

Prove that

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A.$$



L.H.S $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}$

$=\dfrac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$

$=\dfrac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\cos A}$

$=\dfrac{1+\sin^2 A+\cos^2 A+2\sin A}{(1+\sin A)\cos A}$

$=\dfrac{1+1+2\sin A}{(1+\sin A)\cos A}$

$=\dfrac{2+2\sin A}{(1+\sin A)\cos A}$

$$= \frac{2\cancel{(1+\sin A)}}{\cancel{(1+\sin A)}\cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2\sec A = \text{R.Hs Proved}$$

Examples 13

Prove that

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta.$$

$$\text{L.H.S} = \frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\operatorname{Cosec}\theta = R.H.S \text{ Proved}$$

Examples 14

Prove that

$$\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}=1+\sin\theta\cos\theta.$$

L.H.S $\dfrac{\cos^2\theta}{1-\dfrac{\sin\theta}{\cos\theta}}+\dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$\dfrac{\cos^2\theta}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}+\dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\cos^2\theta\times\cos\theta}{\cos\theta-\sin\theta}+\dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\cos^3\theta}{\cos\theta-\sin\theta}+\dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\cos^3\theta}{\cos\theta-\sin\theta}+\dfrac{\sin^3\theta}{-(\cos\theta-\sin\theta)}$

$$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos^3\theta - \sin^3\theta}{(\cos\theta - \sin\theta)}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \cos\theta \cdot \sin\theta)}{(\cos\theta - \sin\theta)}$$

$$= 1 + \cos\theta \cdot \sin\theta = \text{R.H.S proved}$$

Examples 15

Prove that

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}.$$

L.H.S $\quad \dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\operatorname{Cosec}^2 A}{\operatorname{Sec}^2 A - \operatorname{Cosec}^2 A}$

$$\frac{\dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\sin^2}{\cos^2 A} - 1} + \frac{\dfrac{1}{\sin^2 A}}{\dfrac{1}{\cos^2 A} - \dfrac{1}{\sin^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A \cdot \sin^2 A}}$$

$$= \frac{\sin^2 A}{\cancel{\cos^2 A}} \times \frac{\cancel{\cos^2 A}}{\sin^2 A - \cos^2 A} + \frac{1}{\cancel{\sin^2 A}} \times \frac{\cos^2 A \cdot \cancel{\sin^2 A}}{\sin^2 A - \cos^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A}$$

$$= \frac{1}{1 - 2\cos^2 A} = R.HS$$

Examples 16

Prove that

$$\frac{\sec^3\theta}{\sec^2\theta-1}+\frac{\operatorname{cosec}^3\theta}{\operatorname{cosec}^2\theta-1}=\sec\theta\operatorname{cosec}\theta(\sec\theta+\operatorname{cosec}\theta).$$

[CBSE 2016]

$$\text{L.H.S} = \frac{\sec^3\theta}{\tan^2\theta}+\frac{\operatorname{cosec}^3\theta}{\cot^2\theta}$$

$$= \frac{\frac{1}{\cos^3\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}+\frac{\frac{1}{\sin^3\theta}}{\frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{1}{\cos^3\theta} \times \frac{\cos^2\theta}{\sin^2\theta} + \frac{1}{\sin^3\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos\theta \cdot \sin^2\theta} + \frac{1}{\sin\theta \times \cos^2\theta}$$

$$= \sec\theta \times \operatorname{cosec}^2\theta + \operatorname{cosec}\theta \times \sec^2\theta$$

$$= \sec\theta \cdot \operatorname{cosec}\theta \left(\operatorname{cosec}\theta + \sec\theta\right) = \text{R.HS Proved //}$$

Examples 17

Prove that

$$\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{1-2\cos^2 A}.$$ [CBSE 2014, '15]

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L.H.S :- $\dfrac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{(\sin A-\cos A)(\sin A+\cos A)}$

$= \dfrac{\sin^2 A+\cos^2 A+2\sin A\cdot\cos A+\sin^2 A+\cos^2 A-2\sin A\cdot\cos A}{(\sin^2 A-\cos^2 A)}$

$= \dfrac{1+1}{1-\cos^2 A-\cos^2 A}$ $\Big|$ $= \dfrac{2}{1-2\cos^2 A}$ = R.H.S Proved

Examples 18

Prove that

$$\frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A + \cot A} + \frac{\operatorname{cosec} A + \cot A}{\operatorname{cosec} A - \cot A} = 2(2\operatorname{cosec}^2 A - 1) = 2\left(\frac{1+\cos^2 A}{1-\cos^2 A}\right).$$

$$= \frac{\operatorname{Cosec} A - \operatorname{Cot} A}{\operatorname{Cosec} A + \operatorname{Cot} A} + \frac{\operatorname{Cosec} A + \operatorname{Cot} A}{\operatorname{Cosec} A - \operatorname{Cot} A}$$

$$= \frac{(\operatorname{Cosec} A - \operatorname{Cot} A)^2 + (\operatorname{Cosec} A + \operatorname{Cot} A)^2}{(\operatorname{Cosec} A + \operatorname{Cot} A)(\operatorname{Cosec} A - \operatorname{Cot} A)}$$

$$= \frac{\operatorname{Cosec}^2 A + \operatorname{Cot}^2 A - 2\operatorname{Cosec} A \cdot \operatorname{Cot} A + \operatorname{Cosec}^2 A + \operatorname{Cot}^2 A + 2\operatorname{Cosec} A \cdot \operatorname{Cot} A}{\operatorname{Cosec}^2 A - \operatorname{Cot}^2 A}$$

$$= \frac{2\,\mathrm{Cosec}^2 A + 2\,\mathrm{Cot}^2 A}{1}$$

$$= 2\left(\mathrm{Cosec}^2 A + \mathrm{Cot}^2 A\right)$$

$$= 2\left(\mathrm{Cosec}^2 A + \mathrm{Cosec}^2 A - 1\right)$$

$$= 2\left(2\mathrm{Cosec}^2 A - 1\right)$$

Now $2\left(2\operatorname{Cosec}^2 A - 1\right)$

$= 2\left[2\dfrac{1}{\sin^2 A} - 1\right)$

$= 2\left[\dfrac{2 - \sin^2 A}{\sin^2 A}\right]$

$2\left[\dfrac{2 - (1 - \cos^2 A)}{(1 - \cos^2 A)}\right]$

$= 2\left[\dfrac{2 - 1 + \cos^2 A}{1 - \cos^2 A}\right]$

$= 2\left[\dfrac{1 + \cos^2 A}{1 - \cos^2 A}\right]$ Proved

Examples 19

Prove that

$$\frac{\operatorname{cosec}\theta}{(\operatorname{cosec}\theta - 1)} + \frac{\operatorname{cosec}\theta}{(\operatorname{cosec}\theta + 1)} = 2\sec^2\theta.$$

L.H.S $= \dfrac{\operatorname{Cosec}\theta(\operatorname{Cosec}\theta + 1) + \operatorname{Cosec}\theta(\operatorname{Cosec}\theta - 1)}{(\operatorname{Cosec}\theta - 1)(\operatorname{Cosec}\theta + 1)}$

$= \dfrac{\operatorname{Cosec}\theta\left[\operatorname{Cosec}\theta + 1 + \operatorname{Cosec}\theta - 1\right]}{\operatorname{Cosec}^2\theta - 1^2}$

$$= \frac{\operatorname{Cosec}\theta\,(2\operatorname{Cosec}\theta)}{\operatorname{Cot}^2\theta}$$

$$= \frac{2\operatorname{Cosec}^2\theta}{\operatorname{Cot}^2\theta}$$

$$= \frac{2\,\frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}}$$

$$= 2 \times \frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{R.H.S}$$

Proved,,

Examples 20

Prove that

$$\left(\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}\right)\left(\frac{\sin A}{1-\cos A}-\frac{1-\cos A}{\sin A}\right)=4\,\text{cosec}\,A\cot A.$$

[CBSE 2016]

L.H.S

$$\left(\frac{\sin^2 A+(1+\cos A)^2}{(1+\cos A)\sin A}\right)\left[\frac{\sin^2 A-(1-\cos A)^2}{(1-\cos A)\sin A}\right]$$

$$\left(\frac{\sin^2 A+1+\cos^2 A+2\cos A}{(1+\cos A)\sin A}\right)\left(\frac{\sin^2 A-(1+\cos^2 A-2\cos A)}{(1-\cos A)\sin A}\right]$$

$$= \left(\frac{\sin^2 A + \cos^2 A + 1 + 2\cos A}{(1+\cos A)\sin A}\right)\left[\frac{\sin^2 A - 1 - \cos^2 A + 2\cos A}{(1-\cos A)\sin A}\right]$$

$$= \frac{1+1+2\cos A}{(1+\cos A)\sin A} \times \frac{1-\cos^2 A - 1 - \cos^2 A + 2\cos A}{(1-\cos A)\sin A}$$

$$= \frac{2+2\cos A}{(1+\cos A)\sin A} \times \frac{2\cos A - 2\cos^2 A}{(1-\cos A)\sin A}$$

$$= \frac{2(1+\cos A)}{(1+\cos A)\sin A} \times \frac{2\cos A(1-\cos A)}{(1-\cos A)\sin A}$$

$$= 2\,\mathrm{Cosec}\,A \times 2\cot A$$

$$= 4\,\mathrm{Cosec}\,A \cdot \cot A = \text{R.H.S Proved}$$

Examples 21

Prove that

$$\frac{1}{(\operatorname{cosec}\theta - \cot\theta)} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{(\operatorname{cosec}\theta + \cot\theta)}.$$

[CBSE 2010, '12, '16, '18, '19]

L.H.S $= \dfrac{1}{\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}} - \dfrac{1}{\sin\theta}$

$= \dfrac{1}{\dfrac{1-\cos\theta}{\sin\theta}} - \dfrac{1}{\sin\theta}$

$$= \frac{\sin\theta}{1-\cos\theta} - \frac{1}{\sin\theta}$$

$$= \frac{\sin^2\theta - (1-\cos\theta)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{(1^2 - \cos^2\theta) - (1-\cos\theta)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{(1+\cos\theta)(1-\cos\theta) - (1-\cos\theta)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta-1)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

RHS $\dfrac{1}{\sin\theta} - \left[\dfrac{1}{\operatorname{cosec}\theta + \cot\theta}\right]$

$= \dfrac{1}{\sin\theta} - \left[\dfrac{1}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}\right]$

$= \dfrac{1}{\sin\theta} - \left[\dfrac{1}{\frac{1+\cos\theta}{\sin\theta}}\right]$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{1+\cos\theta}$

$= \dfrac{1+\cos\theta - \sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{1+\cos\theta - (1^2 - \cos^2\theta)}{\sin\theta(1+\cos\theta)}$

$= \dfrac{(1+\cos\theta) - (1+\cos\theta)(1-\cos\theta)}{\sin\theta(1+\cos\theta)}$

$$= \frac{(1+\cos\theta)\left[1-(1-\cos\theta)\right]}{(1+\cos\theta)\sin\theta}$$

$$= \frac{1-1+\cos\theta}{\sin\theta}$$

$$= \cot\theta = \text{L.H.S Proved}$$

Examples 22

Prove that

$$\frac{1}{(\sec\theta - \tan\theta)} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{(\sec\theta + \tan\theta)}.$$ [CBSE 2013, '14, '15]

L.H.S $= \dfrac{1}{\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}} - \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\dfrac{1-\sin\theta}{\cos\theta}} - \dfrac{1}{\cos\theta}$

$$= \frac{\cos\theta}{1-\sin\theta} - \frac{1}{\cos\theta}$$

$$= \frac{\cos^2\theta - (1-\sin\theta)}{(1-\sin\theta)\cos\theta}$$

$$= \frac{(1^2 - \sin^2\theta) - (1-\sin\theta)}{(1-\sin\theta)\cdot\cos\theta}$$

$$= \frac{(1+\sin\theta)(1-\sin\theta) - (1-\sin\theta)}{(1-\sin\theta)\cos\theta}$$

$$= \frac{\cancel{(1-\sin\theta)}\,\cancel{(1}+\sin\theta\cancel{-1})}{\cancel{(1-\sin\theta)}\cdot\cos\theta}$$

$$= \tan\theta$$

$$\underline{\underline{\text{R.H.S}}} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta}$$

$$= \frac{1}{\cos\theta} - \frac{1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\cos\theta} - \frac{1}{\frac{1+\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta) - \cos^2\theta}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{(1+\sin\theta) - (1^2 - \sin^2\theta)}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{(1+\sin\theta) - (1+\sin\theta)(1-\sin\theta)}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{(1+\sin\theta)\left[1-(1-\sin\theta)\right]}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{(1-1+\sin\theta)}{\cos\theta}$$

$$= \tan\theta$$

$$= \text{L.Hs}$$

Proved

Examples 23

Using the identity $\sec^2 A = 1 + \tan^2 A$, *prove that*

$$\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{(\sec A - \tan A)}.$$

On dividing by $\cos A$ in Numerator and Denominator.

$$= \frac{\frac{\sin A}{\cos A} - \frac{\cos A}{\cos A} + \frac{1}{\cos A}}{\frac{\sin A}{\cos A} + \frac{\cos A}{\cos A} - \frac{1}{\cos A}}$$

$$= \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$$

$$= \frac{\tan A + \sec A - 1}{\tan A - \sec A + (\sec^2 A - \tan^2 A)}$$

$$= \frac{\tan A + \sec A - 1}{-(\sec A - \tan A) + (\sec A + \tan A)(\sec A - \tan A)}$$

$$= \frac{(\tan A + \sec A - 1)}{(\sec A - \tan A)(-1 + \sec A + \tan A)}$$

$$= \frac{1}{\sec A - \tan A} = \text{R.HS}$$

Examples
24

Using the identity $\sec^2\theta = 1 + \tan^2\theta$, *prove that*

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}.$$

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L.H.S $= \dfrac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$= \dfrac{\tan\theta + \sec\theta - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$

$$= \frac{(\sec\theta + \tan\theta)\left[1 - (\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{(\tan\theta - \sec\theta + 1)}$$

$$= \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \text{R.HS}$$

Proved.

Examples 25

Using the identity $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$, *prove that*

$$\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}.$$

L.H.S $= \dfrac{\cot\theta + \operatorname{cosec}\theta - (\operatorname{cosec}^2\theta - \cot^2\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$

$= \dfrac{(\cot\theta + \operatorname{cosec}\theta) - (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}{(\cot\theta - \operatorname{cosec}\theta + 1)}$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta)\left[1 - (\operatorname{cosec}\theta - \cot\theta)\right]}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta)(1 - \operatorname{cosec}\theta + \cot\theta)}{(\cot\theta - \operatorname{cosec}\theta + 1)}$$

$$= \cot\theta + \operatorname{cosec}\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}$$

$$= \frac{\cos\theta + 1}{\sin\theta}$$

$$= \frac{1 + \cos\theta}{\sin\theta} = \text{R.H.S Proved}$$

Examples
26

Prove that

$$\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta.$$

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L.H.S $= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2 \times \tan\theta \cdot \cot\theta}$

$\{\because \tan\theta \times \cot\theta = 1\}$

$$= \sqrt{(\tan\theta + \cot\theta)^2}$$

$$= \tan\theta + \cot\theta = R.H.S \quad \text{Proved}$$

Examples
27

Prove that

$$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\,\mathrm{cosec}\,\theta.$$

L.H.S $= \sqrt{\frac{\sec\theta-1}{\sec\theta+1}\times\frac{\sec\theta-1}{\sec\theta-1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}\times\frac{\sec\theta+1}{\sec\theta+1}}$

$= \sqrt{\frac{(\sec\theta-1)^2}{\sec^2\theta-1}}+\sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}}$

$$\sqrt{\frac{(\sec\theta - 1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta + 1)^2}{\tan^2\theta}}$$

$$= \frac{\sec\theta - 1}{\tan\theta} + \frac{\sec\theta + 1}{\tan\theta}$$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\tan\theta}$$

$$= \frac{2\sec\theta}{\tan\theta}$$

$$= \frac{2 \times \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$= 2 \times \frac{1}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\sin\theta}$$

$$= 2\,\mathrm{Cosec}\,\theta = \text{R.H.S Proved.}$$

Examples 28

Prove that

(i) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

L.H.S $= \sqrt{\dfrac{1+\sin A}{1-\sin A} \times \dfrac{1+\sin A}{1+\sin A}}$

$= \sqrt{\dfrac{(1+\sin A)^2}{1-\sin^2 A}}$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$\sec A + \tan A = R.H.S$$

Examples 28

$$(ii)\ \sqrt{\frac{1+\cos A}{1-\cos A}} = \operatorname{cosec} A + \cot A$$

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L.H.S $= \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}}$

$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$

$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$

$$= \frac{1+\cos A}{\sin A}$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$$

$$= \operatorname{Cosec} A + \cot A = \text{R.HS proved}$$

Examples
29

Prove that

(i) $\sqrt{\dfrac{1-\sin A}{1+\sin A}} = (\sec A - \tan A)$

L.H.S $= \sqrt{\dfrac{1-\sin A}{1+\sin A} \times \dfrac{1-\sin A}{1-\sin A}}$

$= \sqrt{\dfrac{(1-\sin A)^2}{1-\sin^2 A}}$

$$= \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$$

$$= \frac{1-\sin A}{\cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A}$$

$$= \sec A - \tan A$$

Examples 29

(ii) $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = (\operatorname{cosec} A - \cot A)$

L.H.S $= \sqrt{\dfrac{1-\cos A}{1+\cos A} \times \dfrac{1-\cos A}{1-\cos A}}$

$= \sqrt{\dfrac{(1-\cos A)^2}{1-\cos^2 A}}$

$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$= \frac{(1-\cos A)}{\sin A}$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$\Rightarrow \operatorname{cosec} A - \cot A$

$= R.H.S$

Examples 30

Prove that

(i) $\sqrt{\dfrac{1-\sin A}{1+\sin A}} = \dfrac{\cos A}{1+\sin A}$

L.H.S $= \sqrt{\dfrac{1-\sin A}{1+\sin A} \times \dfrac{1+\sin A}{1+\sin A}}$

$= \sqrt{\dfrac{1-\sin^2 A}{(1+\sin A)^2}}$

$$= \sqrt{\frac{\cos^2 A}{(1+\sin A)^2}}$$

$$= \frac{\cos A}{1+\sin A} = \text{R.H.S Proved}$$

Examples
30

$$(ii)\ \sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$$

L.H.S

$$= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A} = \text{R.H.S}$$

proved

Examples
31

Prove that

$$\sqrt{\frac{1+\sin A}{1-\sin A}}+\sqrt{\frac{1-\sin A}{1+\sin A}}=2\sec A.$$

L.H.S

$$=\sqrt{\frac{1+\sin A}{1-\sin A}\times\frac{1+\sin A}{1+\sin A}}+\sqrt{\frac{1-\sin A}{1+\sin A}\times\frac{1-\sin A}{1-\sin A}}$$

$$=\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}+\sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} + \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} + \frac{1-\sin A}{\cos A}$$

$$= \frac{1+\sin A + 1-\sin A}{\cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2 \sec A = \text{R.Hs proved}$$

Examples 32

Prove that

$$\sqrt{\frac{\sec A - \tan A}{\sec A + \tan A}} \times \sqrt{\frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A + \cot A}} = (\sec A - 1)(\operatorname{cosec} A - 1).$$

[CBSE 2015]

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L.H.S

$$\sqrt{\frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}} \times \sqrt{\frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A + \cot A} \times \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A - \cot A}}$$

$$\sqrt{\frac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A}} \times \sqrt{\frac{(\operatorname{cosec} A - \cot A)^2}{\operatorname{cosec}^2 A - \cot^2 A}}$$

$$= \sqrt{\frac{(\sec A - \tan A)^2}{1}} \times \sqrt{\frac{(\operatorname{cosec} A - \cot A)^2}{1}}$$

$$= (\sec A - \tan A) \times (\operatorname{cosec} A - \cot A)$$

$$= \sec A \cdot \operatorname{cosec} A - \sec A \cdot \cot A - \tan A \cdot \operatorname{cosec} A + \tan A \cdot \cot A$$

$$= \sec A \cdot \operatorname{cosec} A - \frac{1}{\cos A} \times \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A} \times \frac{1}{\sin A} + 1$$

$$= \sec A \cdot \operatorname{cosec} A - \operatorname{cosec} A - \sec A + 1$$

$$= \operatorname{cosec} A (\sec A - 1) - 1 (\sec A - 1)$$

$$= (\sec A - 1)(\operatorname{cosec} A - 1) = \text{R.H.S proved.}$$

Examples

33

Prove that

$$(i)\ \frac{\sin\theta}{(1-\cos\theta)} = (\operatorname{cosec}\theta + \cot\theta)$$

R.HS $\operatorname{Cosec}\theta + \cot\theta$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin\theta}{1-\cos\theta} = \text{L.H.S Proved}$$

Examples
33

$$(ii)\ \frac{1}{(\sec\theta - \tan\theta)} = (\sec\theta + \tan\theta)$$

L.H.S $= \dfrac{1}{\sec\theta - \tan\theta} \times \dfrac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$

$= \dfrac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$

$= \sec\theta + \tan\theta$ = R.H.S proved.

Examples
33

(iii) $\dfrac{1}{(\operatorname{cosec}\theta - \cot\theta)} = (\operatorname{cosec}\theta + \cot\theta)$

L.H.S $= \dfrac{1}{\operatorname{Cosec}\theta - \cot\theta} \times \dfrac{\operatorname{Cosec}\theta + \cot\theta}{\operatorname{Cosec}\theta + \cot\theta}$

$= \dfrac{\operatorname{Cosec}\theta + \cot\theta}{\operatorname{Cosec}^2\theta - \cot^2\theta}$

$= \operatorname{Cosec}\theta + \cot\theta$ = R.H.S Proved

Examples
34

Prove that

$$(\sec^4\theta - \sec^2\theta) = (\tan^2\theta + \tan^4\theta).$$

L.H.S $\sec^4\theta - \sec^2\theta$

$= \sec^2\theta\,(\sec^2\theta - 1)$

$= (1 + \tan^2\theta)\,\tan^2\theta$

$= \tan^2\theta + \tan^4\theta$ = R.H.S Proved.

Examples 35

Prove that

$$(\sin^8\theta - \cos^8\theta) = (1 - 2\cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta).$$ [CBSE 2019]



L.H.S :- $\sin^8\theta - \cos^8\theta$

$= (\sin^4\theta)^2 - (\cos^4\theta)^2$

$= (\sin^4\theta + \cos^4\theta)(\sin^4\theta - \cos^4\theta)$

$= [(\sin^2\theta)^2 + (\cos^2\theta)^2][(\sin^2\theta)^2 - (\cos^2\theta)^2]$

As we know that $a^2+b^2=(a+b)^2-2ab$

$$=\left[(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cdot\cos^2\theta\right]\left[(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)\right]$$

$$=\left(1-2\sin^2\theta\cdot\cos^2\theta\right)\left[1\times\left[1-\cos^2\theta-\cos^2\theta\right]\right]$$

$$=(1-2\cos^2\theta)(1-2\sin^2\theta\cdot\cos^2\theta)$$ = R.Hs Proved

Examples
36

Prove that

$$(\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta = 2.$$

$$\text{L.H.S} = \left[(\sin^2\theta)^2 - (\cos^2\theta)^2 + 1\right] \operatorname{Cosec}^2\theta$$

$$\left[(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta) + 1\right] \operatorname{Cosec}^2\theta$$

$$\left[1 \times \{\sin^2\theta - (1 - \sin^2\theta)\} + 1\right] \operatorname{Cosec}^2\theta$$

$$= \left[(\sin^2\theta - 1 + \sin^2\theta) + 1 \right] \operatorname{cosec}^2\theta$$

$$= \left(2\sin^2\theta - 1 + 1 \right) \times \frac{1}{\sin^2\theta}$$

$$= 2\sin^2\theta \times \frac{1}{\sin^2\theta}$$

$$= 2 = \text{R.H.S Proved}$$

Examples
37

Prove that

$$\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1.$$

L.H.S $\sin^6\theta + \cos^6\theta + 3\sin^2\theta \cdot \cos^2\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta \cdot \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)\left[(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta \cdot \cos^2\theta\right]$

$+ 3\sin^2\theta \cdot \cos^2\theta$

$$= (\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta \cdot \cos^2\theta + 3\sin^2\theta \cdot \cos^2\theta$$

$$= (\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta \cdot \cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta)^2$$

$$= 1^2$$

$$= 1 = \text{R.Hs Proved}$$

Examples 38

Prove that

(i) $(\sin^2 A \cos^2 B - \cos^2 A \sin^2 B) = (\sin^2 A - \sin^2 B)$

L.H.S :- $\sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \sin^2 B$

$= \sin^2 A - \sin^2 A \cdot \sin^2 B - \sin^2 B + \sin^2 A \cdot \sin^2 B$

$= \sin^2 A - \sin^2 B = R.H.S$ Proved.

Examples 38

(ii) $(\tan^2 A \sec^2 B - \sec^2 A \tan^2 B) = (\tan^2 A - \tan^2 B)$



L.H.S $= \tan^2 A \cdot \sec^2 B - \sec^2 A \cdot \tan^2 B$

$= \tan^2 A (1 + \tan^2 B) - (1 + \tan^2 A) \cdot \tan^2 B$

$= \tan^2 A + \tan^2 A \cdot \tan^2 B - \tan^2 B - \tan^2 A \cdot \tan^2 B$

$= \tan^2 A - \tan^2 B = $ R.H.S Proved.

Examples 39

Prove that

$$(\tan^2 A - \tan^2 B) = \frac{(\sin^2 A - \sin^2 B)}{\cos^2 A \cos^2 B} = \frac{(\cos^2 B - \cos^2 A)}{\cos^2 B \cos^2 A}.$$

$= \tan^2 A - \tan^2 B$

$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$= \frac{\sin^2 A \cdot \cos^2 B - \sin^2 B \cdot \cos^2 A}{\cos^2 A \cdot \cos^2 B}$

$$= \frac{\sin^2 A\left(1-\sin^2 B\right)-\sin^2 B\left(1-\sin^2 A\right)}{\cos^2 A \cdot \cos^2 B}$$

$$= \frac{\sin^2 A-\sin^2 A \cdot \sin^2 B-\sin^2 B+\sin^2 A \cdot \sin^2 B}{\cos^2 A \cdot \cos^2 B}$$

$$= \frac{\sin^2 A-\sin^2 B}{\cos^2 A \cdot \cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cdot \cos^2 B}$$

$$= \frac{(1 - \cos^2 A) - (1 - \cos^2 B)}{\cos^2 A \cdot \cos^2 B}$$

$$= \frac{1 - \cos^2 A - 1 + \cos^2 B}{\cos^2 A \cdot \cos^2 B}$$

$$\therefore \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cdot \cos^2 B}$$

Hence Proved //

Examples
40

Prove that

$$\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B.$$

L.H.S $= \dfrac{\tan A + \tan B}{\cot A + \cot B}$

$= \dfrac{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}{\dfrac{\cos A}{\sin A} + \dfrac{\cos B}{\sin B}}$

$= \dfrac{\dfrac{\sin A \cdot \cos B + \cos A \cdot \sin B}{\cos A \cdot \cos B}}{\dfrac{\cos A \cdot \sin B + \sin A \cdot \cos B}{\sin A \cdot \sin B}}$

$$= \frac{(\sin A \cdot \cos B + \cos A \cdot \sin B)}{\cos A \times \cos B} \times \frac{\sin A \cdot \sin B}{(\cos A \sin B + \sin A \cdot \cos B)}$$

$$= \frac{\sin A \cdot \sin B}{\cos A \cdot \cos B}$$

$$= \boxed{\tan A \times \tan B} = \text{R.H.S Proved}$$

Examples 41

Show that $(\cos^2\theta - \sin^2\theta) = \dfrac{2\tan\theta}{(1-\tan^2\theta)}$ **is not an identity**



let $\theta = 30$

L.H.S $\cos^2\theta - \sin^2\theta$

$= \cos^2 30 - \sin^2 30$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

$$R.H.S = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\tan 30}{1-\tan^2 30}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}\sqrt{3}$$

$$R.H.S = \sqrt{3}$$

Hence L.Hs $\neq$ R.Hs

$\therefore \cos^2\theta - \sin^2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ is not an

Identity